## Investigating First Year University Physics Students' Ability to Integrate Algebraic and Kinematics Graphs

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#### Abstract

An important tool in the teaching and learning or study of physics is regarded as Mathematics, i.e., it will not be easy to study Physics without the basic knowledge and skills in Mathematics. Mathematics as a "language of science", and is an expected requirement for students to study physics (Redish, 2005). He furthers argues that physicists blend conceptual physics with mathematical skills and use them to solve and interpret equations and graphs. For instance, in kinematics, different aspects from mathematics such as knowledge of functions and the solving of equations are combined with physics concepts. Many introductory physics students perform poorly on the use of mathematical skills and interpretations of graphs in physics. Two possible reasons may be as follows: i. Students lack the necessary mathematical skills needed to solve the physics problems. ii. Students do not know how to apply and relate their mathematical skills in the context of physics. These two possible reasons were investigated in a Masters Research project which probed first year university students' interpretations of graphs in kinematics and in mathematics. This paper used the idea of Beichner's standardized questionnaire on kinematic graphs. From this questionnaire, an equivalent questionnaire was devised in the context of Mathematical equations and graphs. The results of the investigation tend to indicate the deficiencies in the students' mathematical conceptual knowledge as well as in the transfer of mathematical skills that they possess to solve kinematic equations and graphs. New teaching approaches in the introductory physics, a pre-requite for all STEM studies were thus recommended to enhance student performance in this subject.

#### Keywords

Interpretations, kinematics, equations and graphs, mathematical skills, solve, transfer, deficiencies, conceptual knowledge, introductory physics

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#### Introduction

Students seem to lack knowledge or ability to interpret and analyse kinematics graphs using the knowledge of mathematical skills. In this paper the researcher was investigating if introductory and undergraduate physics students lack mathematical knowledge or is it that they are unable to link mathematical skills and knowledge (algebraic functions and graphs) to solve kinematics graphs. The researcher realised this during students' physics laboratory experiments reports.

A lot of studies and research has been done in physics and mathematics separately, but few have been conducted on the mathematical applications, knowledge and skills in solving physics problems. Essentially this paper seeks to integrate the algebraic skills and knowledge in mathematics to solve physics problems, kinematics graphs in this instance. The convention and naming of algebraic variable in kinematics is one investigation this paper will mainly focus on.

Mathematics has been found to be an essential tool in studying physics, i.e., it will be difficult to study Physics without the sound basics of Mathematics. It is even called the "language of science and math-in-physics is a distinct dialect of that language" (Redish, 2005).

Physicists tend to blend conceptual physics with mathematical skills and use them to solve and interpret equations and graphs. For instance, in kinematics, different aspects from mathematics such as knowledge of functions and the solving of equations are combined with physics concepts.

Many introductory physics students perform poorly on the use of mathematical skills and knowledge in the analysis, interpretations and solving of graphs in physics, kinematics in this particular case study.

This paper will seek guidance and report on two possible reasons may be:

- 1. Students lack the necessary mathematical skills needed to solve the physics problems.
- 2. Students do not know how to apply and relate their mathematical skills in the context of physics.

These possible reasons were investigated in a Masters Research project which probed first year university students' interpretations of graphs in kinematics and in mathematics

This paper will use the idea of Beichner's (TUGK) standardized questionnaire on kinematic graphs (Beichner, 2011). From this questionnaire, an equivalent questionnaire was devised in the context of Physics and Mathematics equations and graphs, i.e., the concept having a similarity and approach. The questionnaire was administered to first year university Physics to write in total confidentiality as the researcher was also their lecturer in both subject and to ensure that students' response is a true reflection of their understanding. This made sure those students don't have to feel intimidated or threatened for fear of being singled out. It was as anonymous as possible, and their results were interpreted in a general format.

The results of the investigation tend to indicate the deficiencies in the students' mathematical conceptual knowledge as well as in the transfer of mathematical skills that they possess to solve kinematic equations and graphs (Redish, E.F., 2005).

#### Literature review

A given graph can be a summarized and conclusive result of a scientific investigation in an experiment or research. Therefore, it is of vital importance that students should understand what a graph is, its comprehension and the tasks involved with it.

Graphs are also referred as visual representations. They used to organise information and to show patterns and relationships. This information is represented by the shape of the graph (Joyce et al 2008). "Graphs are used as a powerful statistical tool to facilitate pattern recognition in complex data" (Eshach, 2010, Van Tonder, 2010 quoting Chamberset al, 1983). Graphs are hence used to summarize large amounts of information at the same time resolving them into details. Graphing is a skill to be used by both experts and novice (laboratory students) to solve problems. It is a tool for data analysis and interpretation. McDermott et al. (1986) concluded that the ability to draw and interpret graphs is one of the most important skills coupled with the analytical tool in the study of Physics.

Physicists use mathematics as an essential tool in physics problem solving (Redish, 2005). This is an evidence that knowledge and understating of algebraic functions and graphs can be used to analyse, interpret and extrapolate information, in order, to solve kinematics equation and graphs. One of the challenges is the problems students have in finding the slope of graph passing through origin and their inability to interpret meaning of area under graph (Beichner 1996 Project, 2011). The struggle by students in physics is reported to be due its complexities and their inadequacies with skills and knowledge of mathematics (Basson, 2002)

Algebra based physics illustrate how an implicit epistemological curriculum can be analyzed, explicated and evaluated and how students' intuitive epistemology plays a role in their learning (Reddish and Hammer, 2009). Students learn better when they are given a chance to explore their difficulties and reiterate their thinking with one another. How motivated students are, tend to have a positive influence to their studies. Maimane, 2006 argued that motivation plays a role wherein learners are actively involved in their learning and as a result students will clarify and develop learning concepts and skills.

Students also need to be able to recognise that that mathematics cannot be disassociated from physics. To learn physics better and be able to solve its problem, mathematics knowledge and skills play a significant role on the part of student understanding. Cognitive blending also helps to deal with problems students encounter in the integration of mathematical and physical knowledge (Bing and Redish, 2007)

Roth defined two tenets of constructivism as:

- Constructed knowledge instead of transmission from educator to student
- Learning as an adaptive process

#### Aims of study

The study aimed at investigating how first year undergraduate Physics students in the Department of Teacher Education at Central University of Technology, Free State, Bloemfontein campus are able to integrate Mathematics and Physics concepts in the presentation and interpretation of kinematics graphs. The problem statement of the study was to investigate whether first year Physics students are able to transfer their knowledge on graphs in Mathematics to graphs in Kinematics.

#### **Research Questions**

- What mathematics knowledge related to interpretation of Mathematical linear functions and graphs do first year Physics teacher students have,
- To what extent do the teacher students apply their mathematics knowledge in kinematics?
- Do they use rote knowledge in the physics questions or show a proper understanding of physics knowledge?

#### Methodology

The possible reasons were being investigated in the researcher's Masters Research project which was has now been completed, which probed first year university students' interpretations and presentation of graphs in kinematics using linear functions and graphs in mathematics. The researcher used a mixed method approach, a quantitative and qualitative research method made of close-ended questionnaire and follow-up interviews.

A standardized questionnaire, structured in similar manner to that, of Beichner, consisting of kinematic and linear function and graphs was prepared by researchers and completed by 1st year Physics teacher students

After students completed the questionnaire, from a group of twelve (12) students, only six were interviewed (explained below) to support their answers, to probe their reasonings in the form of misconceptions and misunderstanding and determine to which extent they applied their mathematics knowledge in kinematics.

Structure of Questionnaire cont.:

- Questionnaire was split into two sections; section A was Mathematics and section B was Kinematics (Physics)
- Questions were mainly on straight line graphs in mathematics and physics
- In Mathematics, students were asked to give/determine gradient, point on a graph, area on a graph
- In Kinematics, students were to determine velocity or acceleration, total displacement of velocity-time graph
- In Kinematics, students were determining instantaneous velocity, or uniform or average velocity or acceleration

- Relationships of displacement and velocity, i.e., when is velocity negative or positive, constant or zero
- Relationships of velocity and acceleration, i.e., when is acceleration negative or positive, constant or zero
- What a given graph represents in terms of y-variable as x-variable, which is time is the same in all graphs

#### Table 1. Aspects of graphs assessed (See Appendix attached)

Aspect		Maths (Part A)	Physics (Part B)
	Reading points	3, 6	4.1
Area	Conceptual	1,7	1, 9, 16
	Calculations	5, 9.1	4.2
Gradient	Constant	2	3, 15
	Positive/negative	9.2, 11	2, 4.1
	Changing	8, 10.1, & 10.2	6.2, 7, 8, 12
	Calculation	12.1 & 12.2	4.3, 5, 6.1
Graph transformation/variety		-	10, 11, 13
$Graph \leftrightarrow Equation$		4, 2.3	14, 17

#### Table 2. Knowledge of forms of graphs (See Appendix attached)

Graph form	Meaning	Questions
s-t: horizontal line	No displacement	7, 8, 10
s-t: straight line	Constant velocity/ constant acceleration	3, 7, 8, 10, 11, 13
s-t: parabolic	constant acceleration	8
v-t: horizontal line	Constant velocity/ constant acceleration	10, 11, 13
v-t: straight line	Constant acceleration	10, 11, 13, 14
a-t: horizontal	Constant acceleration	9, 11, 13
a-t: straight line	Changing acceleration	9, 11, 13

## Relating examples from Questionnaire (See Appendix attached): Section A: Mathematics

From the Figure 1 below, the gradient is:



Figure 1. Straight line graph

(A) Zero (B) Increasing (C) Decreasing (D) Constant (E) Varying

## Relating examples from Questionnaire cont. (See Appendix attached): Section B: Physics

Figure 2 below is a graph of an object's motion. Which sentence is the best interpretation of this graph?



Figure 2. Position vs. time straight line graph

- (A) The object is moving with a constant, non-zero acceleration.
- (B) The object does not move.
- (C) The object is moving with a uniformly increasing velocity.
- (D) The object is moving with a constant velocity.
- (E) The object is moving with a uniformly increasing acceleration.

# Relating examples from Questionnaire cont. (Maths section) (See Appendix attached):

Position versus time graphs for five objects are shown in Figure 3. All axes have the same scale. Which of the following best represents a straight-line graph?



Relating examples from Questionnaire cont. (Physics) (See Appendix attached):

Which of the following represents a straight-line equation when the variables given in brackets are plotted and all other quantities are constants?

(A) 
$$v^2 = u^2 + 2as$$
 (v versus s)  
(B)  $v = \frac{s}{t}$  (v versus t)  
(C)  $v = u + at$  (v versus t)  
(D)  $s = ut + \frac{1}{2}at^2$  (s versus t)

(E) None of the above

#### **Participants**

The study was conducted from available sample of first year physics students, aged mainly between 18 and 22 years at Central University of Technology, Free State (CUT). The study was piloted with a group of 31 first- year undergraduate students at CUT doing physics mathematics. The questionnaire was verified for reliability and validity with the subject head (Senior Lecturer, Physics) and correction were made before final questionnaire was administered. The questionnaire was thereafter administered to 63 participants, who voluntarily completed it. A selected sample of 12 participants, a focus group, from those who completed the questionnaire, was used to conduct interviews to probe their reasoning in the questionnaire responses. Participants' confidentiality was maintained, and they were informed that the result will only be used for research purposes, to improve the teaching and learning of kinematics at first year.

## **Data Collection**

The responses to the questionnaire and focus group interviews were used as collected data. The researcher marked the questionnaire completed by students and used it as data. Responses of the follow-up interviews were collected from and conducted with a selected sample few of participants who undertook the questionnaire. The purpose of the interviews, as discussed below, was to probe the reasoning behind the participants' choices in the questionnaire responses. Data collected was sent to the institutional statistician for data analysis. Biography of the participants was not considered in the data collection due to the scope of the research, other it would have lost the focus.

#### **Data Analysis**

A coding scheme, in terms of number of questions, participants and individual participant response per question, was designed from the completed questionnaire and used for statistical data analysis. Prof A Szurbaga, a statistician, from Institutional Development (CUT) assisted with the statistical analysis. Learners biography was not used in the survey.

Analysis of results was conducted using effect sizes because of its practical significance of these results, since the study was done with available sampling (Lakens, 2013). Cohen's effect sizes (w), given as below, was used to compare proportions' differences in the successes of pilot and post tasks. The results of the survey were statistically analysed using effect sizes since no random sampling was done.

#### Ethical aspects

The completion of questionnaire by students was voluntary. Permission was sought from them to complete the questionnaire with total confidentiality hence students did not have to disclose or write their names on the questionnaire.

#### Results

Of the 87 students, 63 completed the questionnaire voluntarily, the following results were obtained:

Section A was composed of 13 Mathematics questions

- ➢ 69% of students passed section A
- ➢ 5% of Mathematics questions were left unanswered

Section B was composed of 17 Kinematics questions

- ✤ 53% of students passed section B
- ✤ 30% of Physics questions were left unanswered

On average, 56% of students passed the overall questionnaire, as indicated in figure 4 below.

## **Discussion of results**

- Figure 4 shows majority of students fared well in Mathematics questions than in Physics questions
- Students still performed well in Physics but not as good as they did in Mathematics.
- A lot of questions were left unanswered in Physics than in Mathematics by a single student
- Most students left Physics questions unanswered than Mathematics questions
- Students could find the gradient and area easily in Mathematics than in Physics
- Most students have shown that they only learnt to pass their high school and expected to be taught the same thing again in order to pass
- Interviews indicated that though a small fraction used mathematical knowledge, majority attempted to use prior learning and what they have been taught. Participants indicated that they had expected to have taught the topics again before attempting the tasks. This was the highlight or participants' responses to the questionnaire



Figure 4. Bar graph of the pass rate

## Conclusion

There is a clear indication that students can do and answer Mathematics questions with more ease in comparison with Physics questions.

Finding the intercepts, points on the graphs, area, slope and extrapolating information from the kinematics graph posed a greater challenge to participants. They even struggled to identify the type of graph and relate it according to Newton's equation of motion as stated in table 1 and Table 2.

From interviews there were two things observed why students still passed Physics though not as good as they passed Mathematics:

- Most students used their prior and present knowledge in Physics to answer Physics questions than use Mathematical knowledge, i.e., minimal/unnoticed link/use between Mathematics and Physics concepts
- Few students used their Mathematics knowledge and couple it with Physics knowledge to answer Physics questions, e.g., use Mathematics linear graphs to interpret Physics Kinematics graphs

There is still a vast gap of comprehension of previous or prior knowledge and a large number of first year students still expect to be taught what they have learnt at high school in order to apply or use it in the new concepts they learn at university.

A large number of first year students still expect to be taught what they have learnt at high school in order to apply or use it in the new concepts they learn at university.

## Potential implications of findings

The research can lead to an increased realization by Physics lecturers as to how first year Physics students could be helped to interpret kinematic graphs using linear functions and graphs learnt in Mathematics as well as be able to relate Mathematics concepts with Physics concepts.

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#### **Appendix: Research Questionnaire Research Questionnaires SECTION A: Mathematics**

1. Of the geometric figures below, which one has the greatest area? Assume that all the figures have the same width and height.



2. In the graph below, the gradient is:



- Decreasing
- (D) Constant
- (E) Varying

What are the coordinates of the turning point in the graph below? 3.





- 4.1 Which one of the graphs shows the function f(x) = x 1?
  - (A) а
  - (B) b
  - (C) с
  - (D) d
  - (E) e

4.2 Which one of the graphs shows the function  $g(x) = \frac{-}{x}$ ?

- (A) а
- (B) b
- (C) с
- (D) d
- (E) e

4.3 Which one of the graphs shows the function  $h(x) = x^2 - 1$ ?

- (A)
- (B) b

а

- (C) c
- (D) d
- (E) e
- 5. What is the area of the graph below for 0 < x < 3?



- 6. In the graph in question 5 above, how does the value of y change between x = 4 and = 6?
  - (A) Increases
  - (B) Decreases
  - (C) Remains constant
  - (D) Not possible to tell

7. Which of the following has the smallest area under the graph from x = 0 to x = 5?



8. In which of the intervals is the gradient of the graph the largest?





(E) 2 < x < 4

#### 9. Consider the following graph:



9.1 The area under the graph in the x-interval (4, 8) is:

- (A) 0
- (B) 0.75
- (C) 1.33
- (D) 4.0
- (E) 12.0
- (F) 24.0

## 9.2 The gradient of the graph in the x-interval (4, 8) is:

- (A) 0
- (B) 0.75
- (C) 1.33
- (D) 4.0
- (E) 12.0
- (F) 24.0

10. Consider the graph of a function shown in the figure below.



In each one of the questions 10.1 to 10.2 the intervals are labeled as follows:

a :	-4 < x < -2
b :	-2 < x < 0
c :	0 < x < 1
d :	1 < x < 2
e :	2 < x < 4
f :	4 < x < 7

10.1 In which of the intervals is the function increasing?

- (A) c, d, e, f
- (B) c, d
- (C) a, d, e
- (D) Only d

10.2 In which of the intervals is the gradient positive?

- (A) **c**, **d**, **e**, **f**
- (B) c, d
- (C) a, d, e
- (D) Only d







12.3 The equation of the given graph is:





11.1 x = 2? (A) 0.4

- (B) 2.0
- (C) 2.5
- (D) 5.0
- (E) 10.0

## 11.2 x = 4.5?

(A) 0
(B) 8 / 4.5
(C) 4.5 / 8

(D) 8

#### 12.1 The gradient of the given graph is:





## **SECTION B: Physics**

1. Velocity versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in position during the interval?



2. The figure below shows a velocity-time graph of an object's motion. When is the acceleration negative?



3. Below is the graph of an object's motion. Which sentence is the graph's best interpretation?

- (A) The object is moving with a constant, non-zero acceleration.
- (B) The object does not move.
- (C) The object is moving with a uniformly increasing velocity.
- (D) The object is moving with a constant velocity.



4. An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below.



4.1 From 4 to 8 seconds the elevator,

(A) Stands still(B) Moves with constant velocity(C) Moves with increasing velocity(D) Accelerates uniformly

4.2 How far does it move during the first three seconds of motion?

- (A) 0.75 m
  (B) 1.33 m
  (C) 4.0 m
  (D) 6.0 m
- (E) 12.0 m

4.3 What is its acceleration during the first three seconds?

(A) 0.75 m/s2 (B) 1.33 m/s2 (C) 4.0 m/s2 (D) 6.0 m/s2 Time (E) 12.0 m/s2

Position

0

5. The velocity at the 2 second point in the position-time graph below is most nearly:



6. The motion of an object traveling in a straight line is represented by the following graph.



- 6.1 At time = 65 s, the magnitude of the instantaneous acceleration of the object was most nearly:
  - (A) +1 m/s<sup>2</sup> (B) +2 m/s<sup>2</sup>
  - (C)  $+9.8 \text{ m/s}^2$
  - (D)  $+30 \text{ m/s}^2$
  - (E)  $+34 \text{ m/s}^2$

- 6.2 The graph shows that for the three parts of the motion, the object's
  - (A) acceleration first decreased, then remained constant and finally increased
  - (B) velocity first decreased, then remained constant and finally increased.
  - (C) acceleration increased all the time, but with varying amounts
  - (D) velocity first increased, then remained constant and finally decreased
  - (E) acceleration first increased, then remained constant and finally decreased
- 7. The Figure below shows a position-time graph of an object's motion. Which sentence is a correct interpretation of the motion of the object?



- (A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
- (B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
- (C) The object is moving at constant velocity. Then it slows down and stops.
- (D) The object doesn't move at first. Then it moves backwards and then finally stops
- (E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.
- 8. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues with a constant velocity. Which of the following graphs correctly describes this situation?



9. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?



10. The following is a position-time graph for an object during a 5 s time interval.



Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?





- (D) IV only
- (E) V only
- 12. Position versus time graphs for five objects are shown below. All axes have the same scale. Which object has the highest instantaneous velocity in the interval shown?



14. Which one of the following equations correctly describes the line graph?



14

15. Below is a graph of an object's motion.



Which sentence is the best interpretation?

- (A) The object is moving with a constant acceleration
- (B) The object is moving with a uniformly decreasing acceleration.
- (C) The object is moving with a uniformly increasing velocity.
- (D) The object is moving at a constant velocity.
- (E) The object does not move

16. If you wanted to know the distance covered during the interval from t = 0 s to t = 2s, from the graph below you would:



- (A) Read 5 directly off the vertical axis.
- (B) Find the area between that line segment and the time axis by calculating  $(5 \times 2)/2$ .
- (C) Find the slope of that line segment by dividing 5 by 2.
- (D) Find the slope of that line segment by dividing 15 by 5.
- (E) Not enough information to answer.
- 17. Choose the answers of questions 17.1 to 17.4 from the following graph forms.





- (A) a
- (B) b
- (C) c
- (D) d
- (E) e

15

17.2 What is the form of the v versus s graph if  $v^2 = u^2 + 2as$  is plotted with u and a positive constants?

(A) a
 (B) b
 (C) c
 (D) d
 (E) e

17.3 What is the form of the s versus t graph if  $s = ut + \frac{1}{2}at^2$  is plotted with u and a as positive constants?

(A) a
 (B) b
 (C) c
 (D) d
 (E) e

17.4 What is the form of the v-t graph if  $v = \frac{s}{t}$  is plotted with s a positive constant?

(A) a (B) b (C) c

(D) d

(E) e