**IJPCE**
**International Journal of Physics and Chemistry Education, 12(2), 19-33, 2020**

**www.ijpce.org**
**ISSN: 2589-8876**

# Student Understanding of Emergent Aspects of Radioactivity

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## Abstract

In this paper, we report on a pilot interview and subsequent survey study investigating student understanding of radioactivity, particularly half-life. Our findings are consistent with other studies in physics education research, for example, that some students think that an individual atom decays over a prolonged period of time, with half of it being gone at the half-life. We see this naïve idea as a failure to recognize the emergent nature of the decay (that is, a large collection of atoms has different properties than an individual atom does). Research of naïve ideas in radioactivity generally treats the ideas as being stable misconceptions. In this paper, however, we present evidence that student reasoning can fluidly shift when thinking about radioactivity, depending on the context.

## Keywords

emergence, half-life, radioactivity, student understanding

◆Received 4 May 2020 ◆Revised 29 May 2020 ◆Accepted 29 May 2020

## Introduction

Ionizing radiation is utilized around the world for energy, industrial, and medical purposes. However, common use of a technology does not mean that it is commonly understood. In this paper, we present findings of our research on the understanding of junior high school and high school students on radioactivity, particularly regarding half-life. The naïve ideas we observed, as we will discuss below, suggest that students have a tendency to attribute characteristics of a radioactive sample to a single radioactive nucleus. In the case of half-life, there is a tacit assumption that, since the sample decays continuously at a predictable rate, so too must the individual nuclei comprising that sample. We hypothesize that a reason for the popularity of this assumption is that the correct explanation requires ideas of probability and randomness, which are difficult for students to grasp. To situate our work, we begin with a thorough overview of student naïve ideas in radioactivity that have been discussed in literature.

### State of the literature

- Students struggle in understanding emergent phenomena, especially when randomness is involved at the agent level.
- Students struggle in understanding radioactivity, thinking, for example, that an individual radioactive nucleus is half gone after one half-life.
- Naïve ideas of students in physics are not always intact and context-independent misconceptions; sometimes student reasoning shifts fluidly from one idea to another idea that seemingly contradicts the first.

### Contribution of this paper to the literature

- We make the argument that student difficulties in radioactivity may stem in part from their difficulty with emergent and random processes in general.
- Interviews with students reveal that, consistent with what has been found for other branches of physics, student difficulties regarding radioactivity can be context-sensitive; they are not necessarily rigid and robust ”misconceptions”.
- Informed by these interviews, we created and administered a free response survey. Results of this survey similarly indicate that student reasoning about radioactivity can be context-dependent.

### *Student naïve ideas regarding radioactivity*

In describing radioactivity and the interactions of nuclear radiation with matter, there are several technical terms that sound similar and are frequently conflated by students. To say that something is “radioactive“ means that the nuclei of atoms in the object are unstable and that, in the process of losing energy to become stable, “radiation“ will be released. This emitted radiation takes the form of subatomic particles with mass (alpha particles, beta particles, etc.) and high-energy photons (gamma and x-ray). When something is exposed to this nuclear radiation, the object is “irradiated“. Finally, if some of the radioactive material adheres to or is absorbed by the object, then the object becomes “contaminated“. For many learners, these words are severely conflated. Students fail to distinguish radioactive substances from the radiation they emit and consequently think that plumes of radiation arose from the destruction at Chernobyl and were carried by the wind (Alsop, 2001; DeKay & Maidl, 2012; Eijkelhof, 1990; Eijkelhof, Klaassen, Lijnse, & Scholte, 1990; Henriksen & Jorde, 2001; Johnson & Hafele, 2010; Johnson & Maidl, 2014; Millar, 1994; Millar & Gill, 1996; Millar, Klaassen, & Eijkelhof, 1990; Neumann & Hopf, 2012, 2013; Riesch & Westphal, 1975). Of greater practical significance is that students also do not clearly distinguish between “irradiation” and “contamination”. In fact, many mistakenly assume that exposure to nuclear radiation makes objects and people radioactive themselves[i] (Alsop, 2001; de Posada & Ruiz, 1990; DeKay & Maidl, 2012; Eijkelhof, 1990; Eijkelhof et al., 1990; Hafele, 2012; Henriksen & Jorde, 2001; Johnson & Hafele, 2010; Johnson & Maidl, 2014; Millar, 1994; Millar & Gill, 1996; Millar et al., 1990; Prather & Harrington, 2001; Sesen & Elif, 2010). Many are reluctant to eat irradiated food, out of fear that the radiation has made the food radioactive. The erroneous idea that nuclear disasters cause survivors to become radioactive has tragically led to discrimination against Fukushima residents and evacuees (“Fukushima child evacuees face menace of school

https://doi.org/10.12973/ijpce/20529

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bullies," 2017) just as it did (and continues to do) against survivors of the Hiroshima and Nagasaki atomic bombs and their descendants ("Who Are The Hibakusha? | Hibakusha Stories," n.d.). This view that radiation makes things radioactive is prevalent even amongst students in medical school (Kaczmarek, Bednarek, & Wong, 1987; Mubeen, Abbas, & Nisar, 2008). Mubeen *et al.* found that 60.8% of 112 students in their final years of medical school responded on a survey that the room becomes radioactive after an x-ray has been performed (Mubeen et al., 2008). In short, people tend to treat radioactive substances as carrying something like a disease that can spread to other objects and people. As discussed above, there are various types of nuclear radiation, each with different properties and, consequently, different effects on the human body. However, learners often do not distinguish between alpha, beta, and gamma particles, treating them collectively as "bad stuff" (Eijkelhof, 1990; Rego & Peralta, 2006). In fact, many learners do not distinguish ionizing radiation from other types of "bad stuff", like pesticides (Henriksen & Jorde, 2001).

In addition to the "disease" naïve idea just described, research has found other naïve ideas prevalent in learners as well. Generally, students tend to over-simplify risk analysis when radioactivity is involved (Eijkelhof, 1990). Many are unaware that nuclear radiation and radioactive nuclei are ubiquitous[ii], and consider radioactivity and nuclear radiation to be unnatural and/or dangerous things (Alsop, 2001; Boyes & Stanisstreet, 1994; Eijkelhof et al., 1990; Henriksen & Jorde, 2001; Neumann, 2014; Neumann & Hopf, 2012; Rego & Peralta, 2006). Other students, swayed perhaps by classroom demonstrations showing that nuclear radiation is all around us, take the opposite stance, that nuclear radiation is completely harmless. Many students tend to underestimate the risk from natural radioactive sources--like radon gas--and from nuclear radiation used in medicine, and to overestimate the risk from nuclear power plants (Alsop, 2001; de Posada & Ruiz, 1990; Eijkelhof, 1990; Eijkelhof et al., 1990; Henriksen & Jorde, 2001; Millar, 1994; Mubeen et al., 2008; Neumann & Hopf, 2012, 2013; Slovic, 1996). For example, it is common for students to mistakenly think that nuclear power plants can explode with the same devastating effects as a nuclear bomb (Eijkelhof et al., 1990) or to attribute damage caused by nuclear disasters as being the cause of climate change or the depleting ozone layer (Boyes & Stanisstreet, 1994; Neumann & Hopf, 2012). At the same time, others overestimate the risks of nuclear medicine as well, thinking, for example, that nuclear material used in medicine is potentially explosive (Mubeen et al., 2008).

Likely adding to the tendency to overestimate risks from radioactive sources, students tend to lack a clear understanding of how one can protect oneself from nuclear radiation. In practice, shielding is used to attenuate nuclear radiation, with the radiation level decreasing continuously as shielding thickness increases. Many, however, often think about the situation overly simplistically and assume that either all of the radiation is reaching us or none of it is (Eijkelhof et al., 1990; Riesch & Westphal, 1975). Regarding how to block the nuclear radiation, naïve ideas include the notions that radiation can be reflected by a screen, that it can be stopped by a vacuum, and that it can be stopped by counter-radiation (like two colliding jets of water) (Eijkelhof et al., 1990; Riesch & Westphal, 1975).

As mentioned above, nuclear radiation is emitted when unstable nuclei stabilize. However, Nakiboglu and Tekin (Tekin & Nakiboglu, 2006) found that students who learn about nuclear reactions in chemistry class conflate this idea with other mechanisms that transfer energy. For example, many students think that atomic stability and nuclear stability are the same thing, and hence that radon, being a noble gas, cannot be radioactive. After learning that an increase of temperature can increase chemical reaction rates, students assume that temperature affects the rate of radioactive decay as well. Rather than describing radioactivity in terms of interactions between the particles that make up the nucleus, students often think that nuclear energy is released when molecules, atoms, or nuclei collide with each other (Cros, Chastrette, & Fayol, 1988). Other students have been documented in literature who think of nuclear radiation as arising from nuclear particles coming into contact with the atmosphere (de Posada & Ruiz, 1990).

Just as there is confusion regarding how nuclear radiation arises, researchers have documented several naïve ideas regarding the time it takes for radioactive nuclei to decay. Although the point at which an individual nucleus fissions is taken to be random, there is nevertheless a characteristic time necessary for half of a large number of a certain type of radioactive isotope to decay (the "half-life"). For a single atom, this correspondingly implies that there is a 50% chance for fission to occur in a time period of one half-life, provided, of course, that the nucleus has not yet fissioned at the start of that time period. Several studies have demonstrated students applying the concept of half-life incorrectly to individual nuclei, viewing the nucleus itself as being half-decayed after one half-life (Jansky, 2019; Klaassen, Eijkelhof, & Lijnse, 1990). It is also common for students to think that after one half-life, the radioactive sample is no longer dangerous, or even that it is no longer present (Eijkelhof et al., 1990; Lijnse, Eijkelhof, Klaassen, & Scholte, 1990). One study found that students think that a radioactive sample loses half of its mass after one half-life (in fact, the daughter nuclei, which remain in the sample, are essentially the same mass) (Prather, 2005). Anecdotally, there are also students who over-apply the half-life concept, thinking that fuel in a car, for example, has a half-life, and that the amount of fuel in the tank as time passes will decay exponentially (Johnson, 2017).

Unlike naïve ideas in mechanics, which can often be traced readily to everyday experiences of students (*e.g.*, Champagne, Klopfer, & Anderson, 1980) and hence have a visual and/or kinesthetic nature, it has been suggested that the naïve ideas related to radioactivity stem from the large amount of media exposure given to nuclear reactors and weapons (Eijkelhof, 1990; Lijnse et al., 1990).When students learn in the news that there is concern about nuclear weapon proliferation and that nuclear bombs have devastating explosive power, it is little wonder that they adopt the idea that things which are nuclear, including nuclear samples used for medical treatment, can explode (Mubeen et al., 2008). In the Netherlands, for example, cows were allowed to resume grazing after one half-life had elapsed after Chernobyl (Eijkelhof, 1990). It is quite natural that people hearing this on the radio would think that there is no longer any danger after one half-life has passed. While such a misunderstanding may have little practical consequence for long-lived isotopes where the half-life is billions of years, it is an issue relevant to society when dealing with, say, Iodine-131, which was released during the disasters at Chernobyl and Fukushima nuclear power plants and which has a half-life of 8 days.

Regarding why the naïve ideas persist despite instruction, we can envision at least two aspects that make it difficult for students to reach a normative understanding of radioactivity. First, both radioactive nuclei and the radiation emitted are invisible to the human eye. This aspect can help explain the "disease" image of radiation, which does not distinguish between radioactive material and the radiation itself. Specifically, since neither the source of the radiation nor the radiation itself is visible, it should not be surprising that students group the two together. Indeed, efforts to make the particle nature of radioactive material and radiation visible to students, for example, through computer simulations, have demonstrated improved student differentiation between radiation and radioactive material (*e.g.*, DeKay & Maidl, 2012; Johnson & Maidl, 2014). A second plausible reason for the persistence of naïve ideas regarding radioactivity is that the key processes involving nuclear radioactivity are stochastic in nature (Eijkelhof, 1990). This latter aspect, which has not yet been carefully explored, is the focus of this paper and of our research in general.

## Radioactivity is stochastic and emergent in nature

The phenomena of nuclear decay and radiation-matter interactions are predictable in nature, provided a sufficiently large number of events occur. The decay of a single nucleus and the ionization of a single DNA molecule, in contrast, are taken to be random. Students, however, seem to attribute the characteristic of predictability to the individual components as well, thinking, for example, that half of a radioactive nucleus remains after one half-life. We find it likely that students struggle in this way because these situations exhibit *both* randomness *and* predictability, a combination that education researchers in mathematics (Buechter, Hussmann, Leuders, & Prediger, 2005; Gougis et al., 2017; Stavrou, Komorek, & Duit, 2003) and non-linear dynamics (Stavrou & Duit, 2014) have demonstrated to generally be difficult for students. For example, as described by Stavrou and Duit, students tend to think either that a situation is random or that it is predictable, but not both simultaneously (*e.g.*, Stavrou & Duit, 2014). We presume that a student with such a difficulty might hone in on the fact that there is a half-life and, perhaps implicitly, make an argument like the following to argue that a radioactive sample is completely safe after one half-life:

> *If there exists a construct to describe the time evolution of the radioactive sample (namely, half-life), then this is something which can be predicted. However, predictable things are not random. Therefore, it is also predictable how long it will be until there is no danger at all. That required amount of time must be the half-life.*

A similar process of treating nuclear decay as being random at the expense of considerations about predictability could lead to the opposite extreme that nuclear radiation is always dangerous and should be avoided, regardless of how many half-lives have passed, because "you never know with radioactivity – anything could happen".

Situations involving radioactivity exhibit *emergence*, where the characteristic of predictability is found at the macroscopic level (how many nuclei will remain after some time for a large collection of radioactive nuclei, for example) but not at the microscopic level in the *agents* comprising the collective. In general, education researchers have shown that emergent phenomena are difficult for students to understand. Research has been conducted on student understanding of mechanical waves, for example, where each bit of string or air has different properties than the emerging pulse (*e.g.,* Linder & Erickson, 1989; Maurines, 1992; Wittmann, Steinberg, & Redish, 1999). It has been found that students tend to think that wave pulses bounce off each other instead of passing through each other (Wittmann et al., 1999). Molecules of air or of rope, such as those which comprise the waves, generally have the ability to collide and bounce off each other, but the emergent patterns of the waves themselves do not. Also demonstrating a failure to distinguish between agent properties and emergent properties, many students think that "sound" is a characteristic of each individual bit of string or air. Accordingly, many students think that, since sound moves from source to receiver, so too must the air be moving in this way (Linder & Erickson, 1989). Chi and colleagues have argued that much of physics consists of emergent phenomena, where behavior of individual microscopic agents (for example, random collisions of molecules between two bodies at different temperatures) gives rise to a macroscopic process (heat transfer via conduction, predictably driven by temperature differences). Students tend to attribute the characteristics of one scale to that of the other scale, incorrectly thinking, for example, that individual molecules are hot and that each one deterministically delivers a bit of heat to a colder molecule (Chi, 2013).

Wilensky *et al.* uses the phrase "level confusion" to account for naïve ideas such as these (Wilensky & Resnick, 1999). In looking at emergent situations such as evolution and slime mold aggregation where randomness at the agent level gives rise to predictable patterns at the system level, Wilensky *et al.* found that learners tend to repress the randomness in the agent level by utilizing a so-called "deterministic mindset" (Wilensky & Resnick, 1999). Our research investigates the difficulties students have in learning about the emergent aspects of radioactivity, and the research question underlying our study is "what difficulties do learners have in navigating from the stochastic description of individual nuclei and bits of radiation to the emergent picture of radioactivity?" Borrowing Wilensky's terminology, we are interested in documenting what kind of "level confusions" that learners exhibit, as well as in exploring the stability of these confusions. In the next sub-section, we will elaborate on this question by discussing the theoretical background that surrounds it.

## Theoretical background

There is disagreement in the field of physics education research regarding the cognitive structure of student naïve ideas, and, consequently, what the process of conceptual change entails (DiSessa, 2009, 2017; Hammer, 1996b, 1996a; Hull, Jansky, & Hopf, 2020; Redish, 2014; Vosniadou & Skopeliti, 2014). One account considers naïve ideas to be misconceptions that arise from daily experiences and are intrinsically stable. As such, they are obstacles to learning that must be removed before effective learning can take place (Carey, 1986, 2009; Strike & Posner, 1982). With such a view, it follows that the most effective way to deal with naïve ideas is to have students confront them early on, as a sort of vaccine against misinterpreting the knowledge that will be presented in the ensuing classes. Such accounts which consider students as having misconceptions are sometimes described as "unitary ", as students are seen as having just one way of thinking about a given situation at a given time. In particular, students are often considered to have the misconception prior to effective instruction and to have the normative conception after conceptual change has occurred. Generally, the naïve ideas pertaining to radioactivity (described above) are treated in the literature as misconceptions.

An alternative account, described in contrast as "manifold", considers naïve ideas to generally be much more fluid. This perspective treats students as having access to manifold ideas, which shift fluidly depending upon subtleties in the context[iii] surrounding the question at hand. This context-dependency is explained via attributing a different cognitive structure to naïve ideas than what unitary accounts do. Whereas the unitary view (usually implicitly) considers the naïve ideas themselves to be the fundamental pieces of cognition, the manifold account considers naïve ideas to be assemblies of smaller pieces of knowledge that in turn come from physical experiences in the world (Smith, DiSessa, & Roschelle, 1994) and that need not be tightly bound together (Hammer, Elby, Scherr, & Redish, 2006). These smaller pieces, which have been referred to as "resources" (Hammer, 2000), "primitives" (DiSessa, 1993; Kapon & DiSessa, 2010), or "facets" (Minstrell, 1992) are not inherently right or wrong (Hammer et al., 2006). For example, many students often erroneously think that a force is necessary for motion, and perhaps go so far as to describe the "force of an object's motion". This is often accounted for as being an incorrect misconception that "motion is caused by force". However, a manifold perspective might account for the student difficulty by attributing its cause to, for example, an inappropriate usage of the primitive "maintaining agency". "Maintaining agency" is a knowledge element used to understand any continuing effect maintained by a cause, like how wood maintains a fire burning (Hammer, 1996b). Since this and other knowledge pieces are not inherently incorrect and since their interaction with each other is relatively fluid, such a knowledge-in-pieces perspective suggests that the best way to help students learn involves capitalizing on the knowledge pieces embedded within the naïve ideas of students that are shared with the expert view of the material (*e.g.,*Hammer, 1996a).

In light of this theoretical background, we posit our research question as:

> *How can we best describe student difficulties in understanding the emergent properties of radioactivity? Are some difficulties stable in nature, like misconceptions? Are other difficulties fluid and context-dependent?*

## Methodology

Investigation into student reasoning about the stochastic and emergent nature of radioactivity began in 2018 with one-on-one pilot interviews of bilingual high school students (age 14-18) in Vienna, Austria. In this phase of our investigation, it was important for us to get a wide range of different impressions on high school students' reasoning. As such, we decided to allow a variety of student participants, and not to require a representative sample. However, all interviewees had received instruction about radioactivity in a prior physics class. In Austria, radioactivity is part of the national curriculum in grade 8 (RIS, 2020). Topics in typical textbooks include alpha, beta, and gamma decay, fusion and fission, and half-life (see, for example, p. 102 of Fürnstahl, Janisch, & Wolfbauer, 2016). All interviewed students were in grade 9 or higher. In this regard, we can consider these interviewees to be roughly equivalent to high school students in the US who have taken a physics class that discussed radioactivity. Local high school teachers with whom the first author had already collaborated encouraged their students to participate in an interview. Despite receiving no financial compensation for their time or extra credit from their teacher, seven students volunteered to participate. The first author interviewed all seven of these students. The interviews were conducted in English by the first author and lasted between thirty and sixty minutes. The interviews were semi-structured, in that prompts were decided ahead of time, but modified or expanded in response to statements made by the interviewee. These interviews were designed to probe student reasoning about radioactivity in a range of contexts and, to that end, the interview prompts spanned a range of situations (Hammer, 1994): they contained problems taken from the literature reported above, typical textbook problems, and problems specifically created for use in these interviews (four problems that are relevant for the analysis in this paper were also used in the surveys and are described in the next section). The problems were individually printed on paper, with the rest of the sheet blank, and interviewees had access to a pen and scratch paper. These interviews were in the think-aloud style (aka "cognitive labs"). That is, the interviewer requested the interviewees to speak aloud as they think through problems (Ericsson & Simon, 1993) and to not worry about whether their answers are correct or not. These students reported in the interviews that they had learned about atomic structure in physics class and that they had learned about half-life in math class.

Robustness of student reasoning was probed via the "bridging strategy" described by Brown and Clement (Brown & Clement, 1989). In the case where a student answered a question (the "target situation", or "target") incorrectly, the interviewer posed an analogous (from the perspective of the authors) situation that the student would most likely find intuitive and *would* answer correctly (the "anchoring example"). In the case where the student, despite reasoning correctly about the anchoring example, continued to answer the target incorrectly, the interviewer then modified the anchoring example to be conceptually closer to the target. These situations that are intermediate between the target and the anchoring example are known as "bridging analogies." Assuming that the student answered the bridging analogy correctly, the interviewer again asked how this new situation is similar or different from the target, and the process repeated until the interviewee reached either a correct answer to the target or a point where additional analogies did not seem promising (Brown & Clement, 1989). Similar to how Brown and Clement asked interviewees to rank how much a given explanation "makes sense", the interviewer in this study asked interviewees to rank how confident they were in various statements.

After each interview, the first author reviewed the interview and identified anything that might be particularly interesting to discuss with the second author. Segments that both authors found interesting were identified, and the first author presented these segments at research group meetings to receive comments and suggestions from other group members. These comments consisted of various explanations to account for the data and plausibility arguments for what the interviewee was thinking at the time of the interview. Based upon these comments and the ensuing discussions, the authors tested and refined arguments concerning the stability or fluidity of student reasoning.

### Survey creation and analysis

Desiring to investigate not only depth of student reasoning but also breadth of naïve ideas, we decided to create a survey. In doing so, our mixed methods approach can be described as "exploratory sequential" (Creswell, 2014). This survey, in addition to other prompts, would include the following four prompts that had provided fruitful data from the pilot interviews. Answers that we coded as reflecting correct understanding follow each prompt.

1) *Have you learned the term "half-life" before? How would you explain it to a classmate who missed class that day?*

A scientifically-acceptable answer would be something like "half-life is the time needed for half of a sample of radioactive atoms to transform into a different type of atom".

2) **"Closet" prompt:** *Radon-222 is an example of a radioactive atom. It has a half-life of about 4 days, meaning if you start with a whole bunch of the atoms, only half of them will still remain after 4 days. Suppose your closet has somehow become filled with this gas, and the door is taped shut so that none of it can escape. Would you feel comfortable removing the tape and opening the closet? Or would you want to wait a while first? How long would you wait?*

A practical answer would be something like "I would wait until the radioactivity level is comparable to that of the air I breathe outside of the closet".

3) **"Many vs 1" prompt, Part 1:** *Imagine that you begin with 100 million Radon-222 atoms. How much Radon-222 will remain after a) 4 days, b) 8 days, and c) 12 days? Explain briefly, how you reached your answers.* **// Part 2:** *Imagine that you begin with a single Radon-222 atom. How much Radon-222 will remain after a) 4 days, b) 8 days, and c) 12 days? Explain briefly, how you reached your answers.*

The correct answer to Part 1 is "50, 25, and 12.5 million atoms" with an explanation like "only half of the atoms remain after 4 days, so you divide by 2 every 4 days". A scientifically acceptable response to Part 2 would be "either 0 or 1 for all points in time, because it is random when an individual nucleus decays."

4) **"Cage" prompt** (Jansky, 2019)**:** *Suppose you have a friend who has just freshly created one of these [Radon-222] atoms and is keeping it in a cage. You really want to see the atom transform, but your parents will only let you take one day off from school to go watch the nucleus. Would you go on the day your friend first created the atom to go watch and see if it transforms? Or would you wait until a later day? Which day?*

We would expect a student who understands half-life as an emergent property to answer "it doesn't matter, because it is random when an individual atom decays".[iv]

The authors translated this survey into German. The first author then conducted survey validation interviews with three additional high school students (age 14-18, with equivalent prior content exposure as the pilot interviews) in German. Like with the pilot interviews, the authors met to discuss the validation interviews, and minor changes to wording of the prompts ensued. Finally, the first author administered this survey in June 2019 to 55 junior high school students (13-14 years old) visiting the University of Vienna. These students took the survey prior to a lesson from pre-service teachers on radioactivity. Before their visit, these students had not yet had any instruction on radioactivity. We justified gathering survey data from younger children than we had in the case of interview data on a number of grounds. First, as will be discussed more fully in the "Results" section below, there seemed to be relatively little knowledge that the older students had retained pertaining to half-life that was useful in answering these prompts. Specifically, although they recalled having learned the concept in math class and having solved problems with it involving exponentials and logarithms, they often did not have a firm grasp on the idea that the decay of a single atom is random. Second, the definition of half-life was provided to survey respondents, at the start of the "Closet" prompt and at the start of the "Cage" prompt (see the "Closet" prompt above).

Once the survey responses had been collected, the authors then analyzed the data in an entirely descriptive manner, coding the responses using qualitative content analysis, beginning with set categories, but adding to the coding scheme inductively (Mayring, 2014). In the first step of this, the first author selected 6 responses to the "Cage" prompt and 6 responses to the second part of the "Many vs 1" prompt (asking about the single atom) that spanned a range of student responses and were hence representative of the 55 responses. The first author proposed coding categories, both for the answer provided by the respondent, as well as for the reasoning accompanying that answer. The authors then discussed these codes and made minor modifications to the coding scheme. Then, each author independently coded the remaining 98 survey responses (49 responses for each of the two prompts). This totaled 196 responses to code (both "answer" and "reasoning" for each of the 98 survey responses). The authors then calculated percentage of (identical codings divided by all codings) (Mayring, 2014). Out of these 196 responses, the authors agreed perfectly on 137 (70%) of them. On 10 of the remaining 59 responses, there was partial agreement (a number of respondents were assigned multiple codes for "reasoning"). The item with the greatest disagreement was the reasoning for the second part of the "Many vs 1" prompt (concerning the single atom). The answer of respondent A7, for example, was "0.5, 0.25, 0.125", which both authors coded as "Level confusion". However, in response to the request for an explanation of where these answers came from, the respondent wrote "I consistently divided by half". Whereas one author coded this reasoning as "Half-gone after T ½", the other author coded this as "None/Irrelevant" as it was a description of the mathematical manipulations performed, not a reasoning to support *why* those manipulations had been performed.[v] After agreeing that responses which *only* describe what mathematical procedure was performed should be coded as "None/Irrelevant", 15 of the 59 disagreements were resolved. The additional disagreements were resolved on a case-by-case basis until full agreement had been reached, with the exception of one response to the explanation for the second part of the "Many vs 1" prompt, which was unclear and could be interpreted in various ways. This response was excluded from the results presented below (see **Tables 2** and **3**).

### *Interview Results*

The seven pilot interviews yielded data supporting the idea that students struggle with emergent aspects of radioactivity. Here, we present as case studies excerpts taken from three of these interviews, focusing on the "Cage" prompt and a graphical form of the "Many vs 1" prompt that we incorporated into the survey. We present results from the survey itself in the next section.

Each of the seven interviewees were asked if they had learned about half-life before, and the answer was unanimously "yes". Some specifically recalled learning about it in math class, and solving problems involving calculating how much of a radioactive sample would remain after some time. Some recalled that exponents and/or logarithms are important. Several of these same interviewees, however, drew a straight line for the first part of the graphical form of the "Many vs 1" prompt (see, for example, the first graph in **Figure 1** below). While this is interesting and potentially indicates difficulties with mathematical sensemaking, investigation of this graphical error is outside of the scope of our current research project. We wish only to note that interviewees *had* studied the concept of "half-life" in class prior to the interviews, but clearly had not mastered it. Of relevance to our study is not this particular graphical error, but that interviewees struggled to recognize half-life as an emergent phenomena, as the following interview analyses will demonstrate.

### *Alex stably views the decay of a single nucleus as continuous and prolonged*

In answering the "Many vs 1" prompt, Alex[vi] drew similar graphs. In explaining the second graph (for the single atom), Alex said "Well, I also think it would go down. I'm not sure how fast it would go. I mean it is only one atom, but... so it would look a bit similar, I guess, only a bit steeper." Referring to the first graph (of the full sample) drawn, Alex continued "Well, if you look at this, it still isn't finished, so I'm going to go with like 50, 30 days it will be gone."

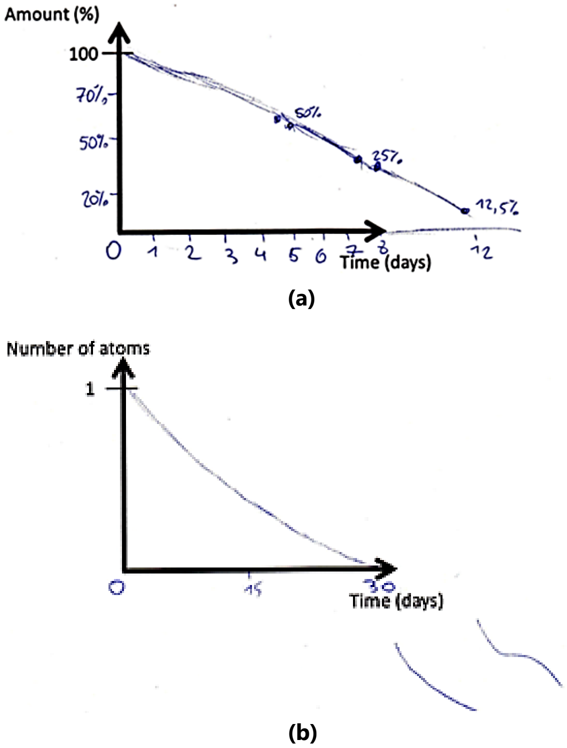


**Figure 1.** Alex's responses to the "Many vs 1" prompt. The two curves in the bottom-right of (b) are in response to the prompt "is there anything else you are thinking it could be?"

It seems that Alex was thinking of individual radioactive nuclei as decaying over a prolonged period with the same order of magnitude of time as the half-life. As discussed above in the literature review, this naïve idea has been documented by other researchers as well (Jansky, 2019; Klaassen et al., 1990). This is evidence to support the argument that Alex was struggling with the emergent aspect of half-life. Specifically, Alex seemed to attribute the continuous decay of the overall sample to the individual atoms. Particularly strong evidence for this is Alex discussing how long the decay of the overall sample takes when drawing the second graph ("Well, if you look at this [first graph], it still isn't finished…")

Having observed this, the interviewer then attempted to determine how stable this view was by means of presenting Alex with an anchoring example. In this anchoring example, the interviewer asked Alex to join him in flipping 100 coins at a rate of one per second[vii]. The interviewer explained that if the coin lands "heads", it is to be put into a "discarded" pile and not used again. If it lands "tails", on the other hand, then it is to be used in the second round. In this second round, Alex and the interviewer would re-flip the (approximately) 50 coins that had landed "tails", but more slowly, such that the total time of the round would still be 100 seconds. After giving this explanation, the interviewer then asked how long it takes for half of the coins to be "discarded", assuming that half are discarded each round. As expected, Alex understood that it would take 100 seconds (the duration of one round). The interviewer then asked Alex how long it takes for any specific coin (a coin painted green in advance of the coin flipping) to be put into the "discard" pile. Again as expected, Alex answered correctly, that there is no way to predict when it will occur, but that it is a process that occurs essentially immediately. The interviewer then asked Alex to graph the decrease in coins as time passed, both for the whole collection of coins, and for the individual coin, and Alex did this correctly as well. These correct answers and corresponding graphs served as an anchoring example (Brown & Clement, 1989). The interviewer then asked Alex how this situation is similar or different to the question with the two graphs involving the Rn-222 (**Figure 1**). Alex replied that although luck is involved with the coin, that is not the case for the decay of the atom. Alex expressed a confidence level of 9 (out of 10) that the single coin goes from one to zero instantaneously, but only a 1 or 2 for the graph of the single atom. Considering that perhaps the reason Alex was lacking confidence with the graph of the single atom was due to consideration that it might be like the single coin after all, the interviewer asked "is there anything else that you think it might be?" Here, Alex did not say that it might be like the coin (an instantaneous process that starts at a random point in time). Rather, Alex's uncertainty seemed to arise from the particular shape of the curve for the single atom. Adding the two curves in the bottom-right of **Figure 1**, Alex responded "It might not be as even, so it might, instead of a line like this, it might go like this. It depends how drastic it breaks down." We see this data as suggesting a difficulty for Alex to consider that, although the collection of radioactive atoms decays over a prolonged amount of time, this is not the case for a single atom. That is, Alex exhibits a "level confusion" in attributing a characteristic of the whole to the individual agent, and this naïve idea seems relatively stable.

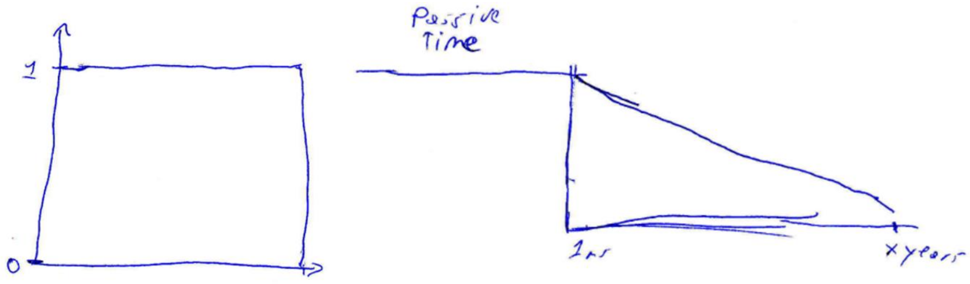
To be clear, we are not arguing that Alex's graph for the single atom is a replica of the graph for the whole sample. Whereas the first graph (incorrectly) appears to be a straight line, the second graph seems to be curved for some reason. At one point when first making the second graph, Alex considered that it should be steeper than the top curve (although Alex seemed to abandon this in having the decay of the single atom last 30 days). In making the revised curve, Alex acknowledged that the decay of a single atom could be an even more complex function than the curve originally drawn. It is not the case, then, that Alex refused to consider that the collective has any properties that are emergent. Our point, rather, is that Alex felt that the decay of a single atom should be a continuous and prolonged process, and that this error indicates confusion regarding emergence ("how can essentially instantaneous processes produce a seemingly continuous one?"). A critic of our analysis could argue that the interviewer led too strongly, "tricking" Alex, by introducing half-life in the "Many vs 1" prompt. Certainly, we acknowledge that Alex may have answered the question differently if the interviewer had not provided the half-life of Rn-222 and had not requested a graph pertaining to a large number of radioactive nuclei. We intentionally designed this prompt, however, to investigate student reasoning of the emergent nature of radioactivity. And here, we see Alex struggling to recognize that half-life is an emergent property, thinking instead that if the collection of atoms undergoes a time-intensive process, then so must the constituents as well. Furthermore, although the idea of half-life was salient in Alex's mind, so too was the emergence-related idea in the anchoring example (although the collection could take an extended amount of time to "go into the bag", a single agent goes in at any time at random and very quickly). Despite this saliency, Alex did not consider that the single nucleus could similarly deviate from the sample in its decay. Without stronger interviewer intervention, the idea that the single atom might have a step function for a decay curve like that of the single coin did not occur to Alex.

It was common for students to first answer the "Many vs 1" prompt with graphs that were similar in some way, as Alex's were. Although for Alex, the bridging analogy seemed to have little, if any, influence on the interviewee's reasoning, this was not generally the case across the seven interviews, as the two following case studies highlight.

### *Chris becomes convinced of the right answer, at least for the moment*

We also have a case of a student, Chris, who becomes convinced of the right answer after the interviewer introduced the coin-flipping analogy. Prior to the analogy, Chris seemed to be torn between two ideas. On the one hand, Chris voiced the idea that the decay of an individual atom occurs in a moment's time. On the other hand, Chris struggled to see how such a behavior could give rise to the emergent behavior of the collection of atoms. Chris seemed to settle with the idea of the decay taking place instantaneously (and drew the graph on the left in **Figure 2**); however, when asked for a confidence rating, Chris answered "I am not confident in anything about this" and pointed out that it could be a slow decay taking a number of years (the second graph in **Figure 2**).

It was at this point that the interviewer led Chris through the coin flipping analogy. Unlike with Alex, the bridging strategy resulted in a "Eureka!" moment in the final minutes of the interview for Chris, with Chris expressing more confidence in the idea of the decay taking place instantly



**(a)** **(b)**

**Figure 2.** Chris's two rivaling solutions for the amount of Rn-222 as time passes when starting with just one atom. Prior to the bridging strategy, Chris was more confident in the correctness of (b) than in the correctness of (a). After the bridging strategy, this was reversed.

(left graph) than in the idea of a slow decay (right graph). Although this was nice to see, we cannot imagine that this is evidence of conceptual change occurring for two reasons. First of all, the change that we see in this interview occurred over the course of minutes, a time span significantly less than that traditionally associated with conceptual change (e.g., Carey, 1986; Strike & Posner, 1982). Secondly, we have evidence that it is *not* the case that the incorrect idea has been uprooted and replaced with the correct conception. At the end of the interview, Chris still expresses some attachment (a confidence of 3 out of 10) to the wrong idea. What's more, even at the beginning of the interview, Chris had access to the correct graph for the single atom (although the confidence rating was much less). In summary, we do not have evidence of conceptual change occurring in this interview. We do, however, have evidence that the naïve idea that the decay of a single atom takes place continuously over a prolonged period, while seemingly rigidly fixed in Alex's mind, is less robust for Chris. This finding is relevant, as it informs analysis of student naïve ideas in our research. That is, it may be the case that a "manifold" perspective of student understanding (e.g., DiSessa, 2017; Hammer, 2000) may be a productive lens through which to view student reasoning about radioactivity. We will return to this point in the Discussion, after presenting the final case study of Bailey, whose ideas about the decay of a single atom were even more fluid.

### *Bailey demonstrates fluidity in reasoning about radioactivity*

Unlike Alex and Chris, Bailey drew an incorrect graph for the first part of the "Many vs 1" prompt and, consequently, was led through the bridging strategy *prior* to drawing the second graph. Specifically, Bailey first drew all of the 100 million radon atoms being gone after two half-lives. As discussed above in the literature review, this and the similar naive idea that a radioactive substance would be gone after *one* half-life have been documented by physics education researchers (Eijkelhof et al., 1990; Lijnse et al., 1990). In response, the interviewer posed Bailey an abbreviated anchoring example, consisting only of "flipping" the coins and drawing a graph for the number of coins remaining as time passes (but not thinking about an individual coin). When asked how the anchoring example compares with the case of atoms decaying, Bailey responded in a way that revealed awareness of the random nature of radioactive decay. To be specific, Bailey said that the flipping of coins is different from the decaying of atoms because "Atoms don't follow a scheme… ten can fall apart at once, and then in the next two seconds, only one". Despite awareness of this randomness, Bailey maintained the view that all of a radioactive sample will predictably be gone after two half-lives, different from the case of flipping coins. As Bailey had identified the salient difference between the anchoring example and the target being that the coins were being flipped one at a time with a steady interval, the interviewer then posed to Bailey a situation of 100 people in a room, each with one coin to flip, at some point of the individual's choosing, during 100 seconds. If the individual's coin lands "heads", then that person must step out of the room. This situation served as a bridging analogy (Brown & Clement, 1989), a situation between the anchoring example and the target.

Bailey said this situation is also different from what occurs with atoms, because whereas people might be left in the room after 200 seconds if their coin keeps turning up tails, no atoms will remain after two half-lives. Bailey's stance resisted even a second bridging analogy that was considerably more leading:

> ***Interviewer:*** *Suppose that you had a friend… who comes up to you and says hey, I have this crazy idea. Um, I think that what atoms do, is they flip a coin. And at some point, in like, [100] seconds, at some point in 4 days, they flip a coin, and if it is heads, then they break apart and if it is tails, then they stay, and they wait until the next four days, and then they flip it again. What would you think about this idea?*
>
> ***Bailey:*** *It's not very likely… Because when I flip a coin, the probability that I get a tails is 50%, but an atom, it has this specific amount of time. After that, it WILL break apart* (emphasis Bailey's)… *It is definite.*

Bailey then specified that, in the case of Rn-222, the "specific amount of time" after which the atom must break apart is four days. Noticing the contradiction between this and the graph Bailey had drawn for the Rn-222 sample (where all of the sample goes to zero after *two* half-lives), the interviewer probed deeper into this idea, first by requesting the drawing of the second graph of the "Many vs 1" prompt, for a single radioactive atom. Bailey drew a step function, specifying that it goes from 1 to 0 at any day at random, but with a maximum time limit of 4 days. Bailey did not seem to notice any contradiction between the two graphs even with questioning from the interviewer, and so at last the interviewer asked directly whether the longest time a single atom can last is 8 days, as shown in the first graph, or 4 days, as shown in the second graph. At this point, Bailey modified the bottom graph such that the atom has a time limit of 8 days. Approximately four minutes elapsed between Bailey first drawing the step function with a time limit of four days and the modification to making it be eight days. It was a considerably awkward period with many terse one-word responses from Bailey, and interviewer dynamics may have been substantial.[viii] Following the resolution that the graph for the single atom should have a time limit of 8 days, the interviewer checked how confident Bailey was, and here Bailey made a surprising shift in thinking.

> ***Interviewer:*** *OK, I understand, I think. Let me check in with you. How comfortable are you on a scale of 1 to 10, 10 being like, super comfortable, yes, it makes good sense, how comfortable are you with this graph [the collection of radioactive atoms]?*
>
> ***Bailey:*** *Now that we've talked about it, not that confident.*
>
> ***I:*** *What else are you thinking maybe it could be, in that case?*
>
> ***B:*** *Well, it could be that, after, like 4 days, then this amount halves. Like, in 8 days, we only have 25% then.*
>
> ***I:*** *Ah... I see. Would that be consistent with other things that we have talked about? Like why are you thinking that that might be the case now? You said, after talking about it, you are no longer so convinced, so what did we talk about that makes you think maybe it could be that instead?*
>
> ***B:*** *Well, we talked about the people and that someone might still have the tails!*

To be clear, it is not surprising that Bailey expressed a lack of confidence. In fact, during the four minutes necessary to transition from a single-atom graph with a four-day limit to one with an eight-day limit, Bailey admitted considerable confusion, remarking "I wish I could understand what I'm saying!" What is surprising, however, is that the lack of confidence now expressed was not about the length of the time limit for the decay process, which had been the focus of the four-minute exchange. Rather, it was about whether there should be a time limit *at all*, an assumption Bailey had firmly clung to throughout the interview. It is possible that, having recognized at last the inconsistency in first saying that the single atom must decay within four days, Bailey's confidence *in general* was weakened. It is further possible that this weakened confidence manifested itself here with Bailey questioning whether or not the atoms are actually like people after all.

Desiring to return to this prompt later in the interview when Bailey was feeling more comfortable in general, the interviewer intentionally deliberated in asking Bailey how, exactly, "atoms being like people" would change the graphs. Instead, the interviewer asked Bailey the "Cage" prompt first. An outline of the relevant parts of Bailey's interview are in **Table 1**.

Bailey said, correctly, that the atom might still be present in the cage, not yet fissioned, even at a point later than two half-lives. When the interviewer finally had Bailey return to the "Many vs 1" prompt, Bailey was consistent in applying this idea to make a new graph not only for the sample of atoms, but for the single atom as well, saying that, assuming that the idea that atoms are like people flipping coins "is correct, which it probably is", then the atom could last for a long time. From this data alone, one might conclude that, by relying upon the bridging analogy of people flipping coins, Bailey has succeeded in coming to understand the emergent nature of radioactive decay. However, it is even less appropriate to consider this to be conceptual change than it was for Chris. Like with Chris, Bailey's change in thinking takes place over the course of an hour-long interview, much less time than typically considered necessary for conceptual change. Like Chris, Bailey did not walk away with 100% confidence in the correct answers. Regarding the idea that the atom in the "Cage" prompt could possibly still be there even 9 days later, Bailey expressed a confidence of only 5 out of 10. Finally, although Bailey was consistent in one regard in answering the "Cage" prompt (namely, in saying that a single nucleus could survive longer than two half-lives), the new context also brought about a shift in reasoning.

**Table 1.** Outline of the relevant parts of Bailey's interview. Despite improvement in responses to the "Many vs 1" prompt, Bailey continued to utilize half-life when reasoning about a single atom in the "Cage" prompt.

| Time | Content |
|---|---|
| 13 min | Bailey draws the Rn sample reaching zero at 8 days |
| 15 min | Anchoring example: graph of sample of coins is exponential |
| 24 min | Bailey: the anchor is different, because "atoms don't follow a scheme" |
| 25 min | Bridging analogies: 100 people flipping coins, some remain for a long time |
| 27 min | Bailey: the bridges are different, because after T ½ , the atom "WILL break apart" |
| 30 min | Bailey draws a step function for a single atom with time limit at 4 days |
| 34 min | Bailey "corrects" graph for single atom to be consistent with that of the sample |
| 35 min | Confidence check: the atoms could be like the people |
| 36 min | "Cage": the atom could be there past 8 days, but most likely to fission at T ½ |
| 44 min | Confidence check: for the "Cage" prompt, most likely to fission at T ½ |
| 45 min | Confidence check: first graph now extends past 8 days |
| 47 min | Confidence check: second graph now is a step function with no time limit |

As a recap so far, at 30 minutes into the interview, Bailey was arguing that the lifetime of a single atom is confined by the half-life (see **Table 1**). Three minutes later, Bailey has shifted the argument to claim that the lifetime of a single atom cannot exceed *two* half-lives. Shortly thereafter, in the "Many vs 1" prompt, Bailey no longer uses half-life to reason about a single atom at all, thinking that, like a person in a room stuck with a coin that keeps landing tails, we cannot predict when the atom will fission. In the context of the "Cage" prompt, however, Bailey reverts to using half-life to reason about the decay of a single atom, claiming that the best time to watch the nucleus decay is just before the half-life:

> ***Bailey:*** *Because the first two days it is unlikely that it is going to break apart, and the fourth is like, maybe if I come on the fourth, it is already broke, it has already broken apart, so on the third day, is like…*

To be clear, Bailey continued to apply the idea that the single atom *can* exist longer than one or even two half-lives (see remaining transcript from this prompt below). The point, however, is that in arguing about the best time to see the atom decay, Bailey was no longer using the analogy of people flipping coins. Rather, Bailey had reverted to basing the answer upon the half-life.

> ***Interviewer:*** *You think it's going to be less likely to break apart on the first two days, but more likely on the third… on the fourth, it might have already broken apart, OK, I got you. Would there be any point in coming on, like, the fifth day, for example, like would that be a reasonable thing to do, do you think?*
>
> ***Bailey:*** *Yeah, it could still be there.*
>
> ***I:*** *OK. It's possible that it might still be there. OK. But, you are thinking more likely it will be there on the third day.*
>
> ***B:*** *Yea*
>
> ***I:*** *OK. How about on, like 100 days later, for example, what do you think about that? (Jokingly) I'm going to wait one month, and then I'm going to come over.*
>
> ***B:*** *Maybe it's entirely gone by then.*[ix]
>
> ***I:*** *Might be entirely gone by then? OK. Is there any chance that it would still be there, after 100 days, do you think?*
>
> ***B:*** *No. 100 days is a long time.*
>
> ***I:*** *A hundred is a pretty long time, right? How about, like 9 days, for example? So the half-life is 4 days, so we have gone 4 days, another 4 days, and then you come on the 9th day, do you think there would be any chance of it being there?*
>
> ***B:*** *Yeah*
>
> ***I:*** *OK, it might be there in that case. But you are thinking that the third day is the most likely, though. Why is the third day the most likely? Can you expand on that a little bit?*
>
> ***B:*** *It will break apart at one point, and on the fourth day, it is like 50/50, so it might already be gone. It doesn't have to, but it might, and on the third day, well like, I don't know, more... maybe then... on the third day, there are like these atoms in the time between the third and the fourth day, like the end of the fourth day, when 50% are gone, it is more likely that they are going to break apart in my opinion.*

Following this exchange, the interviewer introduced Bailey to the second part of the anchoring example (painting one coin green and following it during the coin flipping). Despite understanding that the likelihood of that coin turning up heads was 50% regardless of which round it was flipped, Bailey remained confident that the most likely time to see the single atom fission was on the third day (confidence rating of 7 out of 10).

In summary, Bailey exhibits in this interview the naïve idea that the half-life is a "special day" for individual radioactive atoms. At first, this followed naturally from Bailey's commitment to the idea that all of a radioactive sample would be gone after two half-lives. However, even after exhibiting understanding that this was *not* the case, Bailey continued to treat the half-life as being useful for making predictions[x] about the individual atom. Instead of now being the day before which the atom *must* decay, it was now the day marking the *most likely* time to decay. It hence seems inaccurate to say that the bridging strategy enabled Bailey to replace some misconception about half-life with the correct conception. Rather, it seems more accurate to think of Bailey's reasoning as consisting of many smaller knowledge pieces including ideas about the random nature of radioactive decay and the usage of half-life for making predictions, and to recognize the bridging strategy as having shuffled these pieces around somewhat.

## Survey Results

The data presented and analyzed above, while limited in scope, serves as an existence proof that student reasoning about emergent properties of radioactivity is not always rigid – it can be context-dependent, shifting fluidly depending on the situation. While interviews are the optimal means for exploring the richness of student reasoning, we also desired to see how widespread difficulties with the emergent nature of radioactivity are. Therefore, informed by these interviews, we created and administered an open-ended survey. We found that the difficulties found in the pilot interviews were not idiosyncratic. The results of our analysis are presented in **Tables 2** and **3** below (N = Number, C = Codes, P = Persons).

We should expect that these survey respondents struggle with understanding half-life even more than the interviewees did, as they have received no instruction other than being presented a definition of half-life on the survey. This expectation is consistent with the survey results. Like Alex and Chris, most respondents described the decay of a single Radon-222 nucleus as being continuous and prolonged, such that half of it remained after 4 days (for example, N=31 out of 55 were coded MA1 in **Table 3**). Like Bailey, many respondents used half-life in arguing for the best time to watch the nucleus in the "Cage" prompt (for example, 13 were coded CR1 in **Table 2**). Of more interest, however, is that we were able to discern with this survey a small number of respondents whose reasoning, like Bailey's, was context-dependent, as discussed in the next paragraph.

As Bailey did, three respondents demonstrated inconsistency between the two prompts. Like Bailey, respondent A5 treated the half-life as a "special day" for the single atom in the "cage" prompt, but not for the "Many vs 1" prompt. Specifically, for the "cage" prompt, the respondent wrote "I would go on a later day, because the half-life of this atom is still four days. So I would maybe go on Thursday." For the "Many vs 1" prompt, on the other hand, the respondent wrote that there would be 1 atom at 4, 8, and 12 days, with the explanation "I am not sure if a person can (for example) halve an atom". From these responses, it seems that the respondent is thinking that the atom remains as it is as time passes on the "Many vs 1" prompt, but that something of significance will happen at the half-life time when responding to the "Cage" prompt. Other forms of inconsistency were found as well. Respondent B17 described it as being unpredictable when the atom in the cage would fission: "I would go on the day the atom is created, because one cannot predict (exactly) when it will transform". This idea of uncertainty, however, was absent in

**Table 2.** Survey responses to the "Cage" prompt. An asterisk (*) indicates the desired response.

| Category | N of C | % of C | % of P |
|---|---|---|---|
| **Answer to the "Cage" prompt** | | | |
| CA1: Half-life | 18 | 33 | 33 |
| CA2: A day NOT T 1/2 | 23 | 42 | 42 |
| *CA3: All days are equally good | 0 | 0 | 0 |
| OTHER | 3 | 5 | 5 |
| NONE | 11 | 20 | 20 |
| **Reasoning on the "Cage" prompt** | | | |
| CR1: The fission is continuous, with half gone by T 1/2 | 13 | 21 | 24 |
| CR2: The fission is a process in some other way | 13 | 21 | 24 |
| *CR3: Unpredictable | 4 | 7 | 7 |
| CR4: The fission takes place at T1/2 | 2 | 3 | 4 |
| CR5: The fission occurs soon after creation | 1 | 2 | 2 |
| OTHER | 3 | 5 | 5 |
| NONE | 25 | 41 | 45 |

**Table 3.** Survey responses to second part of the "Many vs 1" prompt. An asterisk (*) indicates the desired response.

| Category | N of C | % of C | % of P |
|---|---|---|---|
| **Answer to part 2 of the "Many vs 1" prompt** | | | |
| MA1: 1/2 ; 1/4; 1/8 | 31 | 56 | 57 |
| MA2: 1/2 ; 0 ; 0 | 5 | 9 | 9 |
| MA3: 1; 1; 1 // 0; 0; 0 // 1; 0; 0 // 1; 1; 0 | 9 | 16 | 17 |
| *MA4: 1 OR 0; 1 OR 0; 1 OR 0 | 2 | 4 | 4 |
| MA5: 111; 55; 27.5 | 2 | 4 | 4 |
| OTHER | 4 | 7 | 7 |
| NONE | 2 | 4 | 4 |
| **Reasoning on part 2 of the "Many vs 1" prompt** | | | |
| MR1: Half-gone after T 1/2 | 19 | 30 | 35 |
| MR2: "Same as the first part of the prompt" | 8 | 13 | 15 |
| *MR3: Unpredictable | 3 | 5 | 5 |
| MR4: Cannot have half an atom | 4 | 6 | 7 |
| MR5: Atoms do not disappear | 3 | 5 | 5 |
| MR6: The atom is all gone in 2*T1/2 | 3 | 5 | 5 |
| MR7: The atom is eventually gone | 3 | 5 | 5 |
| OTHER | 1 | 2 | 2 |
| NONE | 19 | 30 | 35 |

the response to the "Many vs 1" prompt. Specifically, the respondent had first written "1/2 of a Radon-222 atom" next to "4 days", but had crossed it out and written instead "none". The respondent wrote "none" beside "8 days" and "12 days" as well, with the explanation "one does not have 1/2; 1/4; 1/8 of an atom." The reasoning of this respondent to the "cage" prompt suggests an image of the fission taking place at an unexpected time. In answering the "Many vs 1" prompt, however, the respondent seems to now be reasoning using half-life, to argue that some decay of the atom must have taken place within 4 days time and, since "one does not have 1/2 of an atom", it must be that this decay results in the whole atom being gone. In much the same way, respondent A12 used half-life to incorrectly claim that there would be half of an atom after 4 days on the "Many vs 1" prompt, but discussed in the cage prompt how, if one waited too long, the atom would "*probably* already be gone (emphasis ours)". Although follow-up interviews would be necessary to make more substantive claims, it seems plausible that, for at least a small number of students, reasoning about radioactive decay is not a rigid and robust cognitive structure, but rather something that changes depending on the context of which question is being asked.

## Discussion and Conclusion

In this paper, we have presented preliminary data from our study on student understanding of emergent aspects of radioactivity, particularly of half-life. Our research is situated at the intersection of 1) student understanding of radioactivity and 2) student understanding of emergence. Although it is novel in this regard, we build on research findings in each of these two domains. Education researchers looking at student understanding of emergence have found that students struggle particularly when randomness is involved at the agent level. In particular, when the system exhibits predictability as an emergent phenomena, students tend to take on a "deterministic mindset", avoiding the randomness by instead attributing the characteristic of predictability to the agentive level, exhibiting a type of "level confusion" (Wilensky & Resnick, 1999). We have extended these findings in looking at student understanding of radioactivity. In both interviews and surveys, we have found students who struggle with the random nature of the decay of the single radioactive nucleus, ascribing to it instead the predictable nature of the radioactive sample (a prolonged and continuous decay). Similarly, we have also found students who use the concept of half-life as a "special day" for the single atom, either to argue that it must decay prior to the half-life or that it is most likely to decay *at* the half-life.

These findings are consistent with those of other education researchers looking at student understanding of radioactivity. Researchers have already documented that some students think of the radioactive atom as decaying continuously (Jansky, 2019; Klaassen et al., 1990). Similarly, the view that an atom must decay at some point prior to the half-life would imply that all of a radioactive substance is safe after one half-life, an additional naïve idea that has already been documented (Eijkelhof et al., 1990; Lijnse et al., 1990). These former studies of student naïve ideas in radioactivity have tended to attribute stability to the ideas, albeit implicitly. We, however, have documented evidence that the naïve ideas need not always be stable. We have chosen to discuss the three interviews that we did in this paper because they represent a range not only in terms of student ideas in response to the interview prompts, but also a range in terms of how rigidly students clung to their ideas throughout the duration of the interview. Particularly in the case study of Bailey presented above, we see reasoning that shifts fluidly from moment to moment, particularly in response to the bridging strategy. Even without this strategy, however, in going from one survey prompt to a different survey prompt, we see some evidence of student thinking that depends on the context. Education researchers have documented the fluidity of student reasoning in various physics topics, primarily in the field of mechanics (*e.g.*, DiSessa, 1993; Hammer, 2000; Hammer et al., 2006), but also in other fields such as thermodynamics (DiSessa, 2017) and electromagnetism (*e.g.*, Gupta, Hammer, & Redish, 2010). We are the first that we know of, however, to make such considerations in the field of radioactivity. In terms of our research question, *"How can we best describe student difficulties in understanding the emergent properties of radioactivity? Are some difficulties stable in nature, like misconceptions? Are other difficulties fluid and context-dependent?"* we have provided preliminary data to support the argument that student difficulties regarding half-life *are not necessarily* stable, but rather that student reasoning can fluidly shift from context to context.

At this point, our research is only in a fledgling state. We have conducted seven pilot interviews, developed an open-ended survey, conducted three survey-validation interviews to help validate the survey, and then administered the survey to 55 students. We thus cannot make any wide-sweeping claims, for example, about students in other populations. Although our results are not yet generalizable, they begin to describe the varied and sometimes conflicting ideas about emergent phenomena (like half-life) that students may have early in their studies of radioactivity. Our results also inform instrument development, and we next aim to turn our survey into a closed-form two-tier multiple-choice instrument. First, however, we must carefully consider several issues. In particular, it seems that many students understood the instructions "Explain shortly how you arrived at your answers" in the "Many vs 1" prompt to be equivalent to the instructions "show your work" and correspondingly responded tersely "Always divide by 2" or even just "Math." Similarly, many students answered the "Cage" prompt with personal considerations, for example "On a Wednesday, because I don't like Wednesdays." These responses are included under the "None/Irrelevant" codes in **Table 1**. As a result, we have disappointingly little data pertaining to student reasoning on these prompts. Nevertheless, at this stage, we feel that we have sufficient data to argue that at least some students 1) struggle with radioactivity in part because of attributing predictable properties of the radioactive sample to the individual atoms that comprise this sample, and 2) that this difficulty is not necessarily fixed and rigid, but can rather be context-dependent.

## Instructional Implications

Although he does not pursue the idea, Eijkelhof hypothesizes one of the difficulties in student understanding of the effects of nuclear radiation being its stochastic nature. His suggestion for effective teaching is to begin by ignoring the individual nuclei and to just deal with the macroscopic picture:

> *Our fourth recommendation concerns problems which pupils have with microscopic explanations… We recommend giving more attention initially to the macroscopic aspects of concepts… This would allow pupils to develop a basic conceptual structure… without all the complexities introduced by the micro-approach* (p. 183-184, Eijkelhof, 1990).

However, Wilensky *et al.* have found success in improving student reasoning about emergent phenomena, for example predator-prey relationships (Wilensky & Reisman, 1999a) or the spreading of a rumor (Levy & Wilensky, 2008), when students focus on the individual agents. Specific to emergent phenomena in physics, research has shown that students reach a deeper understanding of the Maxwell-Boltzmann distribution (Wilensky, 1999) and of diffusion (Krajšek & Vilhar, 2010) when they consider the rules of and embody the individual gas molecules. Contrary to the suggestion of Eijkelhof, but aligned with the suggestions of Wilensky *et al.*, we suspect that educators can best help students make sense of nuclear radioactivity despite its stochastic nature by focusing on the properties of the individual nucleus, randomness included, and seeing how different properties can emerge when a sufficiently large number of these atoms are present. To this end, the survey we developed, if further validated and improved, could play a role in identifying student difficulties and hence inform instruction. Although discussing a single radioactive nucleus, like discussing a single atom in a gas, may have little direct practical importance to students, the fact that the latter was effective for teaching macroscopic properties of gases (Krajšek & Vilhar, 2010; Wilensky, 1999) suggests that the former may succeed in helping students understand the macroscopic properties of radioactivity. We expect the learning sequence for high school students described in the recent PhD thesis of Jansky, for example, to be a promising start (Jansky, 2019). Rather than beginning with a macroscopic description of radioactivity, Jansky has students first investigate the law of large numbers by looking at the rolling of dice and the flipping of thumbtacks. Finally, she presents a third analogy of simultaneously releasing an array of rubber popper toys that spring into the air at random points in time. After this sequence of analogies, students in her study were better able to relate the fission of a single nucleus to the statistical description of a radioactive sample.

Regarding student understanding of randomness itself, progress has been reported by Shaughnessy and Ciancetta (Shaughnessy & Ciancetta, 2002). On a survey, when considering dual spinners that are divided evenly into a black and white region, students say that there is a 50% chance of having an outcome where both spinners are landed on black. After actually playing the game in an interview setting, students automatically list the sample space (BW, BB, WW, WB) and realize it is a 25% of winning. The authors argue there is a connection between seeing the variability of the outcomes and coming up with the sample space, which enables calculation of the probability. This process of experiencing the outcomes of random processes is similar to the methods utilized by Wilensky *et al.* (Wilensky, 1999; Wilensky & Reisman, 1999b) and by Krajsek and Vilhar (Krajšek & Vilhar, 2010) for learning about emergent phenomena. We suspect that such an experiential approach to education will be crucial for successful learning of radioactivity. Finally, as preliminary data points to the possibility of context-dependency of student reasoning in this field, we suspect it to be fruitful to adopt a "manifold" perspective (e.g., DiSessa, 2017; Hammer, 1996a, 2000) in curricular design. In particular, it might be promising to pursue instruction that involves "modifying the organization and use of prior knowledge" rather than, as is often recommended by "unitary" perspectives, "dismantling and replacing prior knowledge" (Hammer, 1996a). At this point, our ideas for instructional intervention are largely speculative. They will develop, however, with subsequent research that we plan to conduct

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[i] In actuality, this is possible only when objects are irradiated with neutrons or photons of energy well beyond the gammas used in treating food or in medicine.

[ii] C-14, a trace isotope of carbon in the air as well as H-3, a trace isotope of hydrogen in water, are both radioactive, for example.

[iii] These researchers do not necessarily use the word "context" in the sense of "everyday context" like using the example of an electric eel when talking about electricity. Rather, they mean "context" in a more general sense, and this is how we use the word in this paper as well. Hence, we would say that a block sliding down a ramp and a block sliding on a horizontal surface are two different "contexts" to study student reasoning about friction.

[iv] In fact, the correct answer is "as soon as possible". However, we were not interested in whether students understood the mathematics that implicated this result, but rather in whether or not they understood half-life as an emergent process. Although this prompt describes an experiment that has little (if any!) practical relevance to students, it constitutes a thought experiment that is useful for revealing difficulties students have in understanding the emergent nature of radioactivity.

[v] In contrast, both raters agreed that the reason given by respondent A12, "It becomes half the size every four days" should be coded as "Half-gone after T ½".

[vi] All names are pseudonyms.

[vii] In practice, coins were pulled out of a bag and placed in the palm instead of flipping, so as to speed up the activity and reduce noise. Although each "flip" still tended to take a little longer than one second, interviewees did not indicate any difficulty in imagining it to be a constant rate of one per second.

[viii] For purposes of being transparent with our data analysis, we include transcript of these four minutes in the Supplemental Material available at [URL to be inserted by publisher], and a full transcript is available upon request to the first author.

[ix] Although it is only an isolated statement that was not pursued in the interview, the idea of an atom being "entirely gone" suggests a continuous decay, like that described by Alex and Chris, an idea that was notably absent in Bailey's graphs of the single atom. This might further indicate fluidity of Bailey's reasoning.

[x] To be clear, one can make predictions about single radioactive nuclei using the half-life of the isotope. That is, one can predict with 50% confidence, that, if the atom has not yet fissioned at the present moment, it will have done so if we check again in one half-life. This is not, however, how Bailey was using the construct at this point: in saying that the most likely time for the fission to occur is just before the end of the half-life, Bailey incorrectly attributes physical significance to the half-life not as a time span, but as a point in time.

## Declaration of Conflict of Interests

The authors have no competing interests to declare.

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