

The Impacts of Dynamic Geometry Software on Graphing Abilities of Prospective Physics Teachers: GeoGebra Sample

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Abstract

Dynamic geometry software is computer programs which allow one to visualize abstract concepts as figures and dynamic representations on a computer screen. The effective use of such software as classroom materials becomes possible only when teachers and students have the necessary information and skills. This study aims to identify the graphing abilities of prospective teachers and analyze the impacts of Geogebra, dynamic geometry software, on these abilities. With this aim, a study was conducted on prospective teachers taking general mathematics course. In this study, it was observed that prospective teachers had difficulty especially with drawing and interpreting the graphs of trigonometric and rational functions. However, by using Geogebra they were able to draw and recognize the graphs of functions more easily. By means of interviews with prospective teachers and observations during the performance of activities, findings which verified the same conclusion were also obtained.

Keywords: Dynamic geometry software; Graphing abilities; Prospective physics teachers

Introduction

In today's world where almost all opportunities of technology have great impacts on education, using computers as supporting materials has become an indispensable element of education. Consequently, computer-aided instruction has gradually gained importance and been improved in a variety of ways with the help of a great amount of research. Computer-aided instruction (CAI) helps to the teaching and learning of scientific concepts independent of time (Chang, 2001). Computer-assisted instruction helps students to learn outside their schools as well.

As they help to the development of concrete concepts and are easy to operate, computers are also put into the service of students in mathematical science where abstract concepts are frequently used. Wiest (2001) emphasized that computer use in mathematical education should be mainly based on high level cognitive abilities such as researching, reasoning, hypothesizing and generalizing. Computers are used in mathematics to help students hypothesize, test and generalize with the purpose of encouraging a mathematician to see the path he uses while reaching at mathematical results and to develop a unique way of thinking as well as helping students have opinions about these results (Couco & Goldenberg, 1996).

The educational influence of computers on our life is seen through the use of different computer software. While some of this software has been developed with field specific

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features, some of the are used in a number of different disciplines. There are different computer software systems, the different applications of which can be frequently seen in mathematics education as well as in other disciplines. We can group this software into two main headings: Computer Algebra Systems and Dynamic Geometry Software. Computer algebra systems essentially provide algebraic and numeric calculations while dynamic geometry software provides figures and dynamic representations (Allison Lu, 2008). Allison Lu expressed that the software mentioned with these aspects were completely different two mathematical structures. Hohenwarter and Jones (2007) emphasized that “the forms of computer algebra systems help to the development of graphing ability during the visualization of mathematics”. Similarly, dynamic geometry software was developed in a way including the elements of algebraic symbolization in order to make mathematical problems useful on a larger scale. This kind of software is used as a classroom material in mathematics education. Thus, the dynamic and symbolic nature of computers will help students generalize and formalize the links among their intuitive ideas in mathematics, and think more formally in terms of mathematical information (Godwin & Sutherland, 2004).

What is GeoGebra?

Hohenwarter (2006) defined GeoGebra, known as dynamic geometry software, as “a computer based tool which provides active, experimental and exploratory instruction with the help of symbolic links and which offers many opportunities in this respect”. Beyond defining it as dynamic geometry software, Hohenwarter and Preiner (2007) described GeoGebra as a bridge between computer algebra systems and dynamic geometry software. According to this study, Geogebra has combined the ease of dynamic geometry software use and the specific features of computer algebra systems and consequently has become a bridge among mathematical disciplines such as geometry, algebra and analysis. GeoGebra is used for both the development of educational materials and the visualization of mathematical concepts. It also contributes to active and student-centered education by providing mathematical experiments and interactive studies just as in discovery learning (Preiner, 2008). Diković (2009) performed with university students in Analysis course, concluded that the use of GeoGebra had positive effects on the comprehension and knowledge levels of students. Loc and Triet (2014) have opened a training course on how to apply GeoGebra to teaching mathematics.

In addition to its applications in mathematics education, GeoGebra is also applied in several other disciplines. For instance, Mentrard (2011) designed physics experiments with the help of GeoGebra in his study. In these experiments, materials with various topics including “Newton’s Second Law”, “Coulomb Law” and “Law of Reflection” were developed to help students of physics with their laboratory studies.

Hohenwarter and Fuchs (2004) emphasized that GeoGebra was a considerably useful tool, especially for mathematics instruction in secondary education and Hohenwarter (2006) proposed the following suggestions regarding the use of this software at schools:

1. Equations can be visually presented by teachers.
2. GeoGebra can be used as a structural tool. However, this software should not be used totally instead of the traditional method. It should be used as a supporting material to improve the comprehensibility of the traditional method.
3. With GeoGebra, students can organize their own knowledge themselves and learn by discovering.
4. It can be used to support self-directed learning of students (student-centered).
5. GeoGebra is a suitable classroom material for both students and teachers.
6. Different presentations and similar materials can be prepared and used by teachers during the instruction process (teacher-centered).

The Purpose of the Study

In this study, the purpose is to identify the graphing abilities of prospective teachers and analyze the effects of GeoGebra on these effects. In addition to this, ideas related to what extent prospective teachers can make use of this software and ideas about different applications conducted with the software will be developed. Moreover, the difficulties that prospective teachers face while drawing the functional graphics are identified together with a focus on the question whether these difficulties can be eliminated with the help of dynamic geometry software.

Problems

In this study, the answers of the following problems have been sought:

- a) What level is the graphing and interpretation ability of prospective teachers?
- b) What kinds of changes have been observed in graphing abilities of prospective teachers after the application of GeoGebra?
- c) What are the opinions and suggestions of prospective teachers related to GeoGebra?

Methodology

The study was conducted using one of the qualitative research methods, the case study method. Case study is an empirical inquiry that investigates a phenomenon within its real-life context, in which the borders between the phenomenon and the content are not clearly defined and there is more than one evidence and source of data (Yin, 2003).

Sample

The study was conducted on 5 freshmen at the Division of Secondary Science and Mathematics Education in the Department of Physics Education at a state university in Ankara. As the first year students had just studied the topic of “graphing” included in the program of their General Mathematics course, this class was selected as the research group. The purpose of General Mathematics course included in the curriculum of this department is defined as “to introduce the basic concepts of mathematics; to make a detailed analysis of a function with the help of limit, continuity and derivative and help prospective physics teachers acquire problem solving abilities about the applications of mathematics in the field of physics.” Within the frame of the lesson, in-depth applications on graphing are conducted preceded by the introduction of necessary background knowledge.

Initially, a preliminary study was conducted on 26 prospective physics teachers within the scope of the study. At the end of the preliminary study, 12 students who answered all questions were chosen according to their answers to the question. When the papers of the chosen students were analyzed, 5 students less successful than the others in terms of their graphing abilities were included into the study based on voluntariness.

Data Gathering

The research data consist of study materials prepared by the researchers, the video records of the applications created with GeoGebra and the observation notes of the researchers. The study materials were prepared by researchers with the purpose of testing basic graphing abilities and applied to 4 students who did not take part in the research group. Then, they were revised based on feedback from these 4 students. The new versions of these study materials were presented to the field experts and they were given their final form in accordance with suggestions from them. During the application of GeoGebra, each activity of students was recorded and their images were filed as pictures.

At the beginning of the data collection process, five students from the research group were asked to answer four questions from the research paper. The aim was to identify the level of students in graphing functions, and recognizing and interpreting the functions from given graphics. General information about GeoGebra and its applications was presented by the researchers after the students had answered the questions. During this process, the menu tools of GeoGebra (File Menu, Edit Menu, View Menu, Options Menu, Tools Menu and Window Menu) were introduced one by one. Then, three different views of GeoGebra were explained (Algebra View, Graphics View and Spreadsheet View) in Figure 1.

After a short introduction of the interface, the researchers explained how a graphic belonging to an equation could be drawn by using the spreadsheet view and entry bar on GeoGebra. Some necessary functions like changing the units of axes and using the features of graphics were also introduced.

After the researchers had presented necessary information on GeoGebra, the students had the opportunity to have their own experience in front of computers. They were required to draw the graphics of questions using GeoGebra and this practice was recorded on cameras.

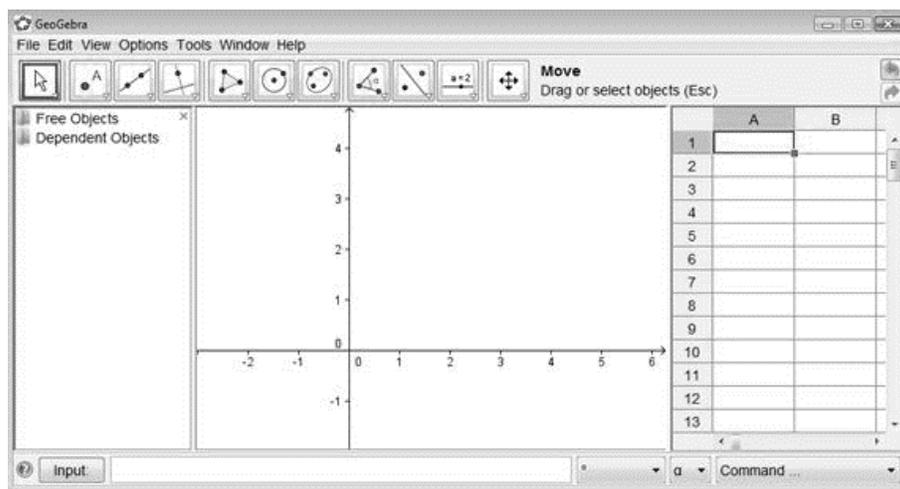


Figure 1. The GeoGebra Interface (Hohenwarter & Hohenwarter, 2009, p. 6)

Data Analysis

The first step of data analysis included the analysis of the students' research materials. The graphics drawn by the students were analyzed, and their prediction of a function from a given graphic and the interpretation of the function's graphic were the main focus of this analysis.

In the second step of data analysis, the recorded images were transferred into the computer while the interviews and monitored images were transferred into a script. From the recorded images, the extent to which the students could use GeoGebra while fulfilling the required tasks was noted. These notes were used to support the answers given during the interviews conducted later.

Findings

Findings Related to the Graphing Abilities of Prospective Teachers

When the papers of the prospective teachers were analyzed, it was noticed that they mainly had problems with graphing trigonometric functions. For example, while graphing the $y = \sin x$ function, they could not identify the curvilinear and the point where the function crosses the axes. The graphic of $y = \sin x$ function and the mistakes students were given in Figure 2.

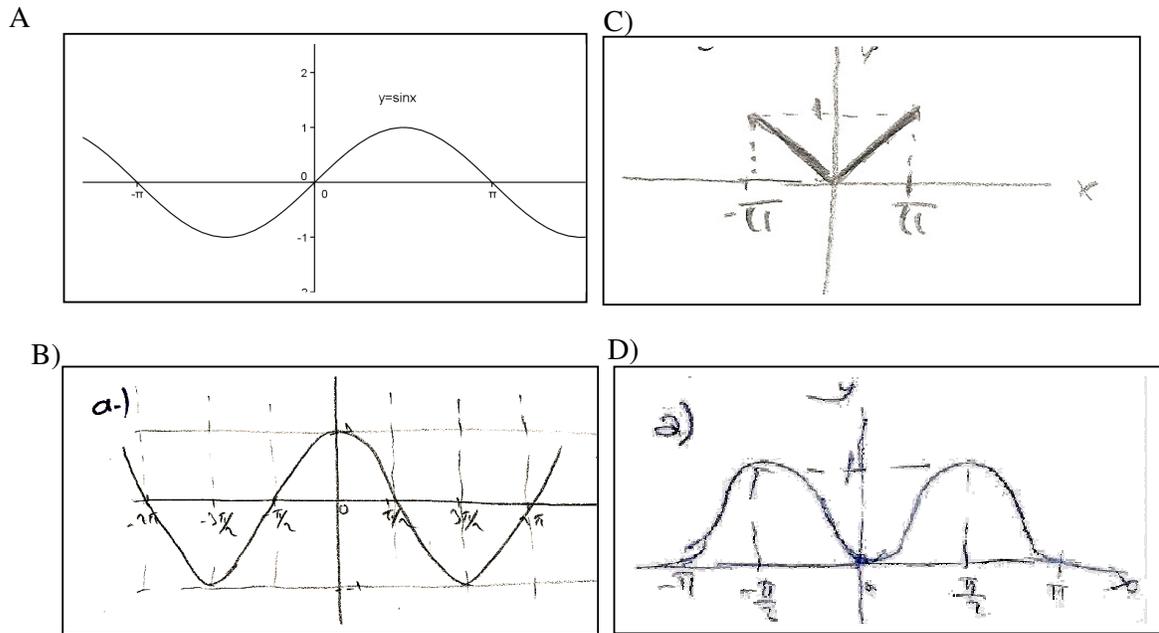


Figure 2. (A) The graph of the given function, (B) The drawing of student number 1, (C) The drawing of student number 3, (D) The drawing of student number 5.

When the papers were analyzed, it was noticed that the prospective teachers were able to draw the graph correctly; however, they were unable to predict the new form of a graph after the application of a variety of changes. For instance, two students who were able to draw the graph of $y = \sin x$ function were required to draw the graphs of functions $y = 2\sin x$ and $y = \sin 2x$. The correct graphs of these functions and examples to the mistakes the prospective teachers were given in Figure 3.

Unlike the first activity, the second activity required prospective teachers to write the function of a given graph. In this activity, the graph of function $y = \tan x$ was given as an example to the prospective teachers. Based on this example, they were required to match the three different graphs with the functions they belonged to. To avoid the chance factor, the participants were given more than three options. The graphs and options presented to the prospective teachers were given in Figure 4.

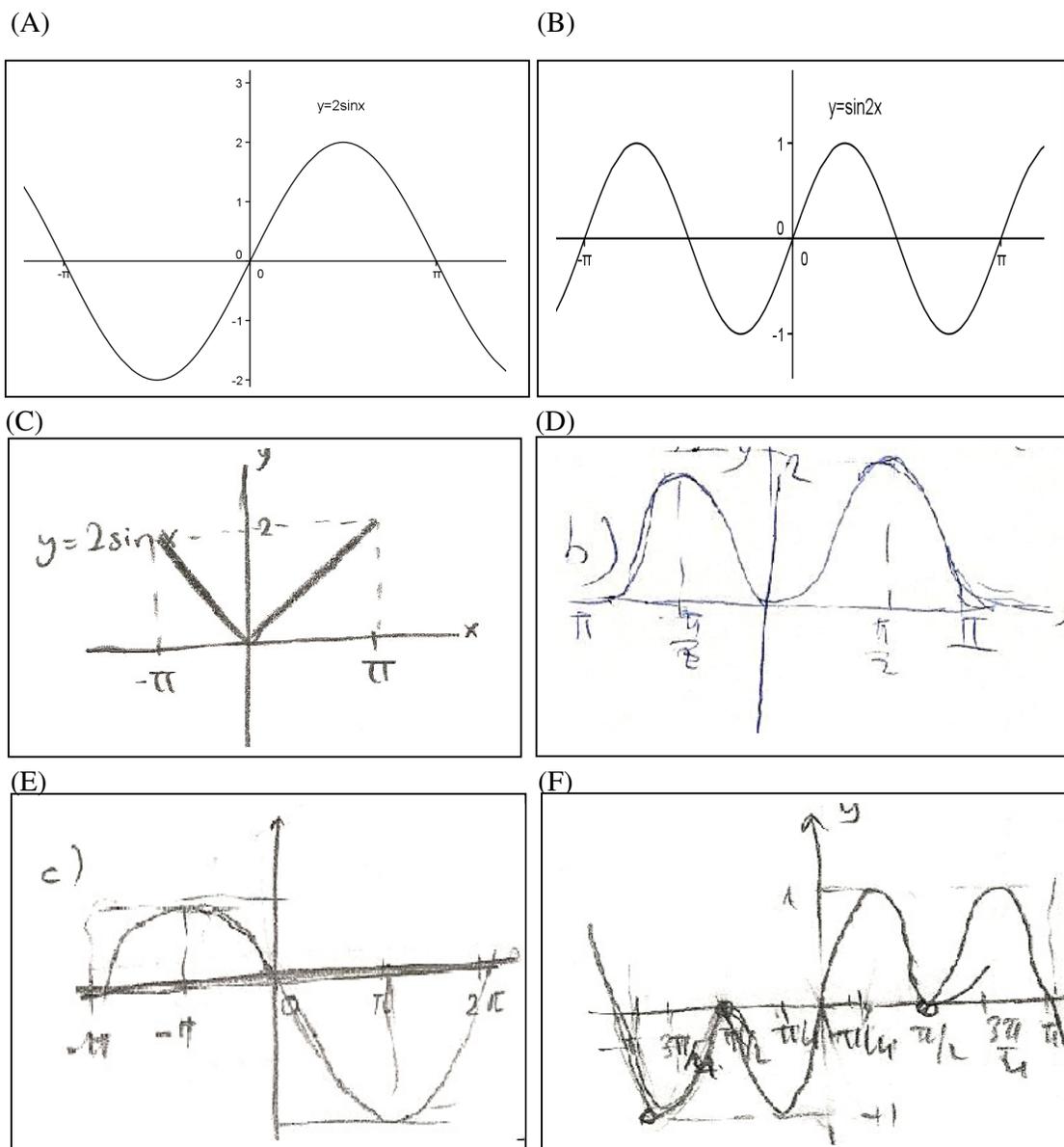


Figure 3. (A) The graph of $y = 2 \sin x$, (B) The graph of $y = \sin 2x$, (C) $y = 2\sin x$ drawing of student number 2, (D) $y = \sin 2x$ drawing of student number 2, (E) $y = 2 \sin x$ drawing of student number 4, (F) $y = \sin 2x$ drawing of student number 4.

The functions given to the prospective teachers were: $y = \tan(x/2)$, $y = \frac{1}{4} \tan x$, $y = \tan 2x$, $y = \tan^2 x$, $y = 2 \tan x$.

When the answers of the prospective teachers were analyzed, it was realized that they had difficulty in predicting the angular alterations of trigonometric functions. For instance, the answer of the first function was $y = \tan(x/2)$, but 4 out of 5 students chose the answer $2 \tan x$ and similarly while the answer of the second question was $y = \tan 2x$, 3 students chose the answer $\tan^2 x$. On the other hand, all 5 students answered the third function correctly.

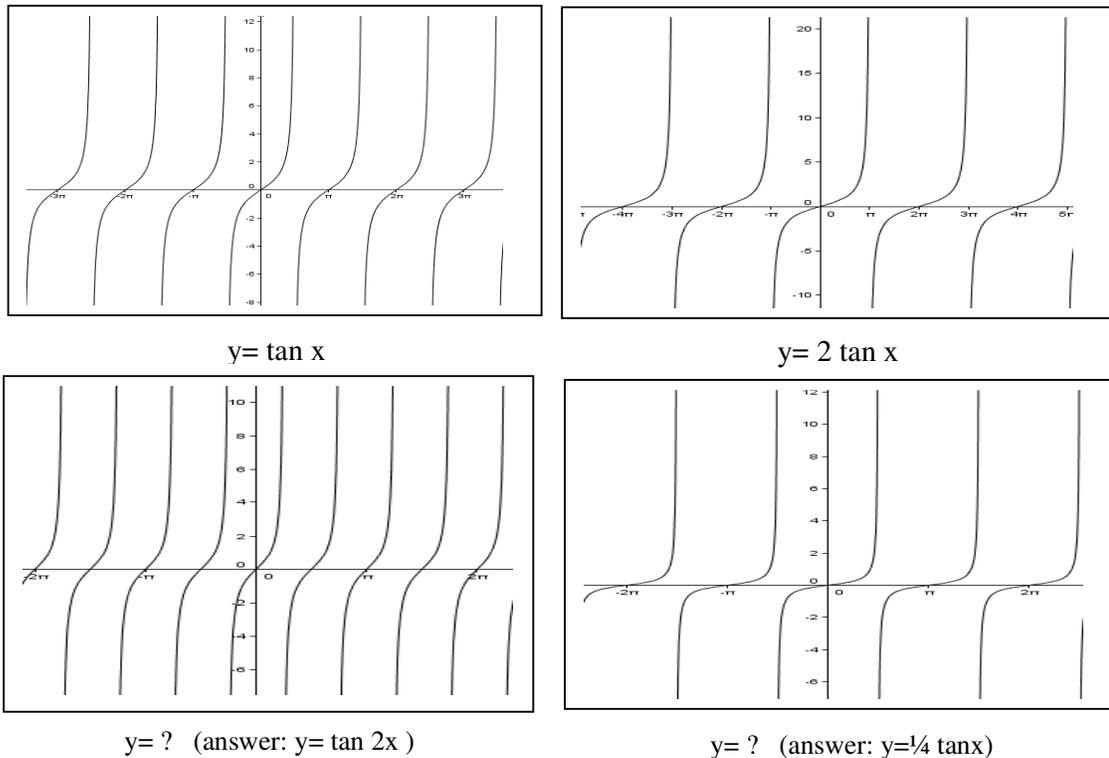


Figure 4. Examples to the questions related to the identification of functions the graphs belong to.

The third activity of the materials focused on the graphic of $y=x^2+3x-4$ equation and interpretation of possible changes that would happen in the graph as a result of some changes in the equation. The prospective teachers were required to answer the following questions:

- * If the coefficient of x^2 is “-3” in this equation, what kind of changes can be observed in the graph?
- * If the coefficient of x is “-2” in this equation, what kind of changes can be observed in the graph?
- * If the constant is 6 in this equation, what kind of changes can be observed in the graph?

The prospective teachers faced some difficulties while drawing the graphs of this equation. First of all, while identifying the point where the graph crossed the axis, they could find the peak coordinates of the parabola, but they marked them as points which crossed the axes, not as the peak point. Moreover, they accepted that the equation was based on a constant after a certain point. The student answers for this activity are presented in Figure 5.

When the answers of the prospective teachers were analyzed, it was concluded that they were not able to predict accurately how the changes that would be done in the functions would be reflected on the graph. For example, the students were asked to answer the question: “what kind of changes will happen when the coefficient of “ x^2 ” is changed into “-3”.The answers were: “the graph shifts 9 units right”, “the x value of radix point increases ‘-1/3’ times”, “the arms of parabola turn downwards and shift “3” units on y -axis”.

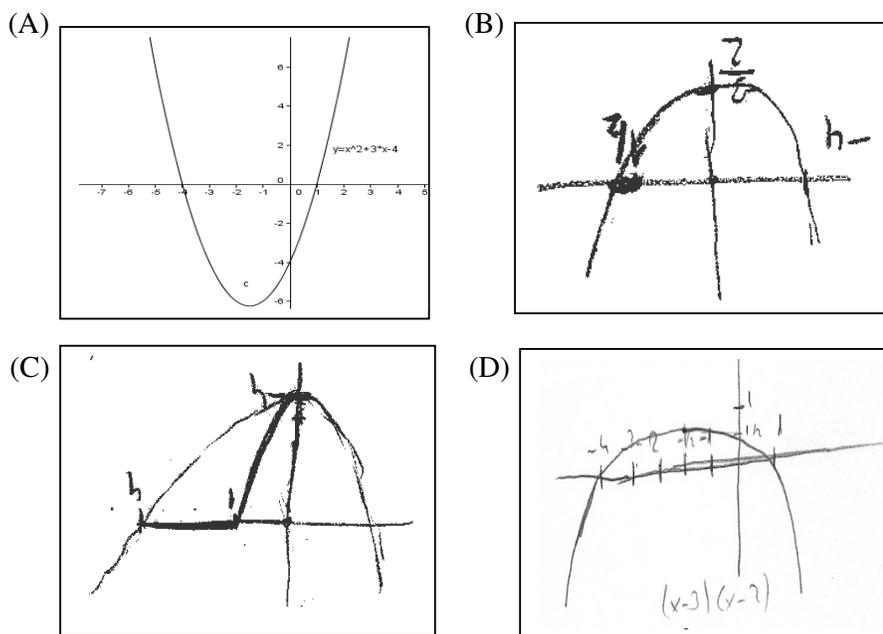


Figure 5. (A) The graph of the given function, (B) The drawing of student number 1, (C) The drawing of student number 2, (D) The drawing of student number 4.

In the last part of the research material, there were questions similar to the previous ones. Unlike the previous ones, the given function equation was rational in this question and it aimed to identify the way the prospective teachers comprehended the concept of asymptote. With this aim, the students were required to draw the graph of $y = \frac{x^2 - 5x + 6}{x - 4}$ function and they were then asked to interpret and draw the new form of the graph when “the coefficient of x^2 is 2”, “the coefficient of x is 4” and “the denominator is $x - 2$ ” in the equation.

When the answers were analyzed, it was noticed that 2 out of five students did not take the asymptote into consideration and consequently drew false graphs. And one student accepted the asymptote as $y = x - 4$ line (Figure 6).

Still similar to the previous question, it was identified that the prospective teachers had difficulty in interpreting the changes that would happen in the graph as a result of changes made in the equation. For instance, the answers to the question which asked “what kind of changes may happen if the coefficient of x^2 is 2 in the equation” were: “the values that the points take change”, “the graph shifts on the y -axis”. In the second item, the answers to the question which asked “what kind of changes may happen in the graph as a result of changes in the denominator of rational expression” were: “the line graph changes into $ax + b$ position”, “the point which the graph crosses the y -axis changes”, “the asymptote changes”, “the points where the parabola crosses the axes change”. The drawings of two students related to this question and their interpretations of changes based on the items are as in Figure 7.

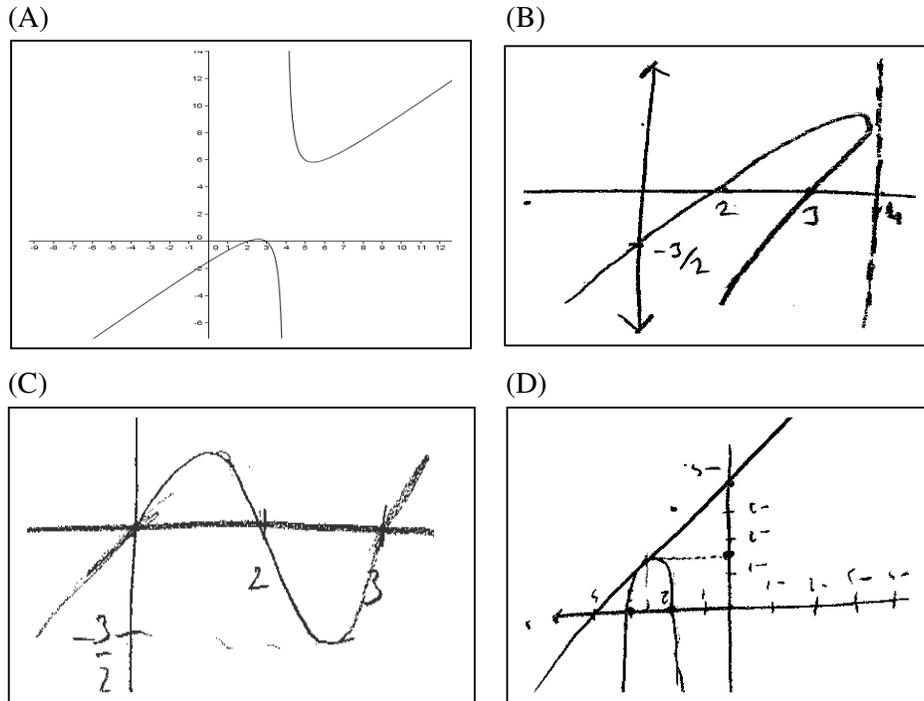


Figure 6. (A) The graph of the given function, (B) The drawing of student number 2, (C) The drawing of student number 4, (D) The drawing of student number 5.

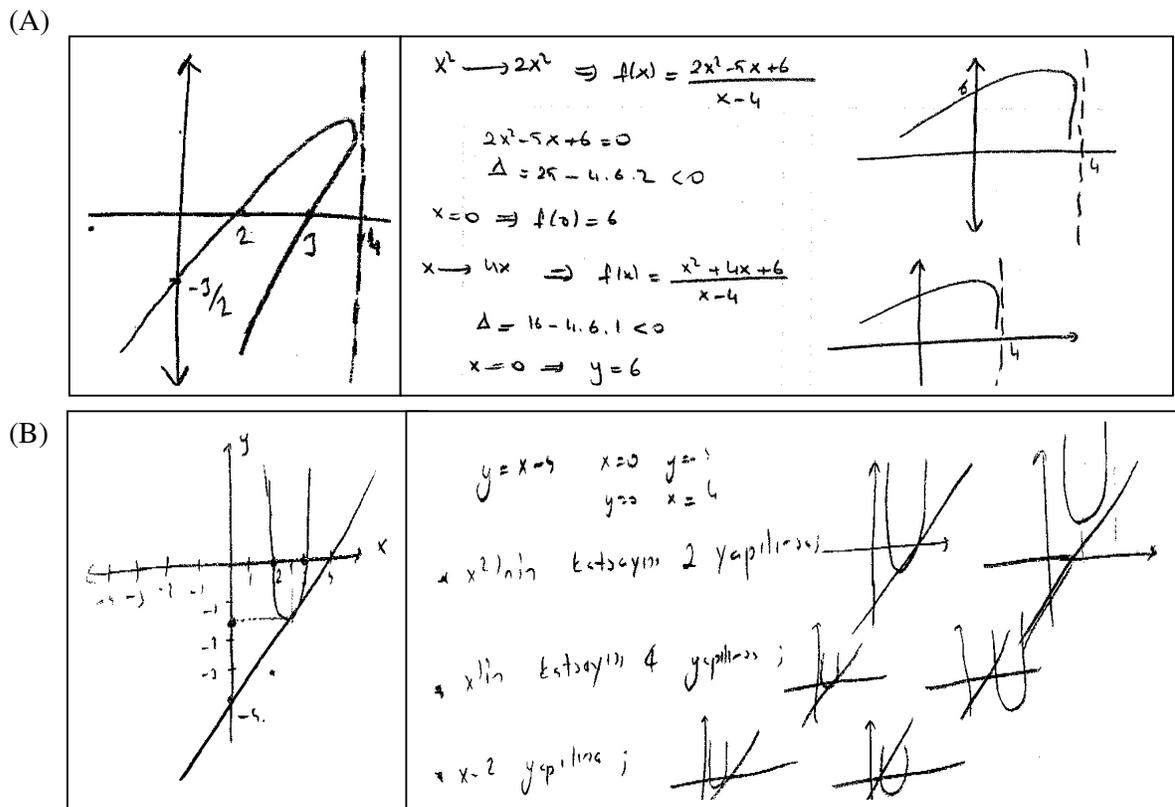


Figure 7. (A) The answer of the student number 2, (B) The answer of the student number 5.

When the Figure 7 is analyzed, it is observed that the student number 2 could identify the asymptote correctly, but he took the function graph only in a pre-asymptote area. Thus, he transferred the changes to the graph inaccurately. Student number 5 similarly took only one area, but different from the other student he accepted the asymptote as a $y=x-4$ curve. This condition could also be similarly noticed in the changes made on the functions. Furthermore, this student expressed that the graph crossed the axes if the coefficient of x^2 was changed into 2. The same condition was also observed when the coefficient of x was changed into 4.

Findings Related to the GeoGebra Practice of Prospective Teachers

GeoGebra helped the prospective teachers to draw the given functions and then observe the changes that would happen in the graphs as a result of changes in these functions.

As a result of this application, the prospective teachers were required to review their answers to the questions which aimed to identify their graphing abilities and then they were asked to make some necessary corrections. The corrections that students did on the first question given as an example are presented in Figure 8. The figures on the right contain the corrections.

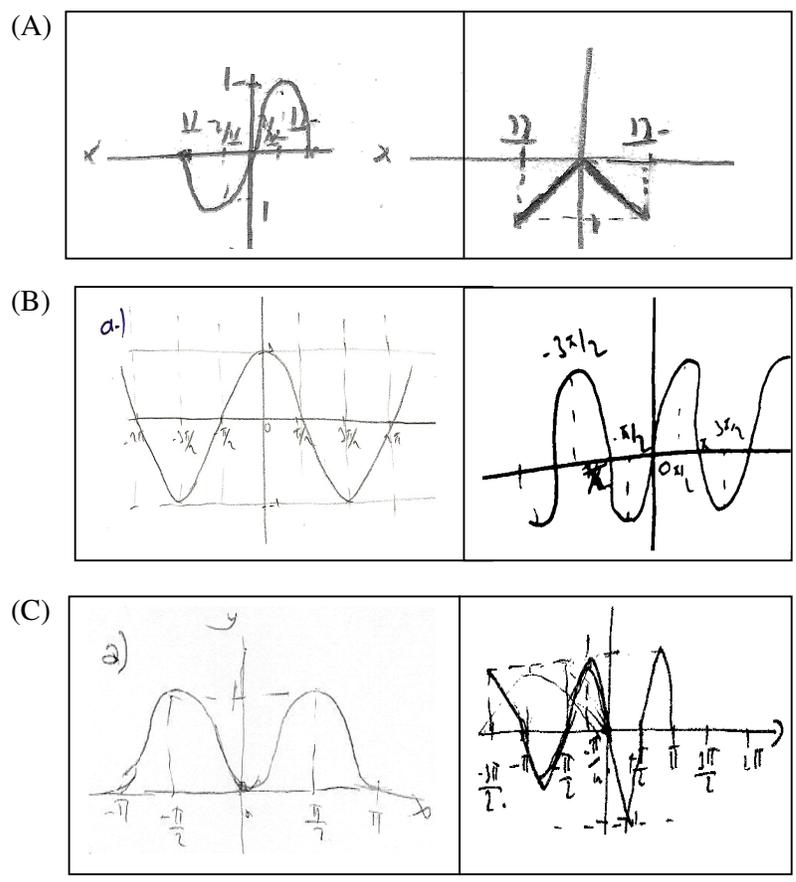


Figure 8. (A) The answer and correction of student number 1, (B) The answer and correction of student number 3, (C) The answer and correction of student number 5.

As it can be observed from the graphs, 3 of the prospective teachers could draw the graphs correctly after some practice with GeoGebra. They could also develop more precise graphics by minimizing the intervals between the points taken on the axes. However, the other two prospective teachers drew inaccurate graphs at the end of their practice with GeoGebra.

When the prospective teachers were asked to re-analyze the second activity in the research materials, they expressed the following ideas:

- While drawing the graph of $\cot x$, I saw that the width of waves increases when the function is $\cot (X/2)$. In other words, the graph of $\cot x$ repeats itself with intervals like $\pi, 2\pi..$, but when the function is $\cot (x/2)$, it repeats itself with intervals such as $\pi, 3\pi, 5\pi...$ Then, the angle in this second graph must become smaller. At first, I thought that the second graph is the graph of $y=\tan 2x$, but when I consider for the second time I can understand that it is the graph of $y=\tan(x/2)$.

- On the drawings I did on GeoGebra, I saw that the graph has more frequent patterns while the angle increases in trigonometric functions. Now, I can understand that my answers were wrong. I think the second graph is $\tan(x/2)$ and the third is $\tan 2x$. In the fourth graph, the intervals on the x-axis have not changed, so I think the angle will not change. But I think there is change in the coefficient. However, I am not sure whether this graph belongs to $y=2\tan x$ or $y=1/4\tan x$.

In the light of these opinions, it was discovered that the prospective teachers could answer the first two graphs of the second question correctly, but they had some hesitations about the answers $2\tan x$ and $1/4 \tan x$ for the last function.

Some of the prospective teachers had difficulty in drawing the graph of the function in the third question and some in interpreting the graph. Thus, a similar version of this question was presented on GeoGebra. The prospective teachers were asked to analyze the graphs they had drawn and the answers they had given before. They were required to think aloud during this analysis and their ideas were recorded. The ideas of the prospective teachers in relation to the graphics of this question were as follows:

- I drew the graphs of similar equations on GeoGebra. When I checked, I saw that the drawing I did was correct. However, I thought that the graph would shift on x-axis when the coefficient of x^2 changed. But now I can see that while the coefficient of x^2 is increasing, the arms become narrow. In other words, the points where the graph crosses the axis change. However, when the coefficient of x^2 decreases, the arms turn downwards and the points where the graph crosses the axis change.

- When we change the coefficient of x , the points where it crosses the axes change. I thought that the graph would become constant after a point. This is the only mistake I did. I used to think that there would be shifts only on y-axis when we changed the constant; however, now I think that the points where it crosses the axis will also change.

- In the drawing I did, the arms of the parabola were downwards. I realized that this was wrong when I thought about it later. However, I realized that the changes in coefficients would in any case change the points where the graph crossed the axes. I thought that it would only change for the coefficient of x^2 . Now, I understand that all of my answers are wrong.

The following drawings for the third question show the first and final versions of graphs drawn by the prospective teachers before and after this thinking process in Figure 9.

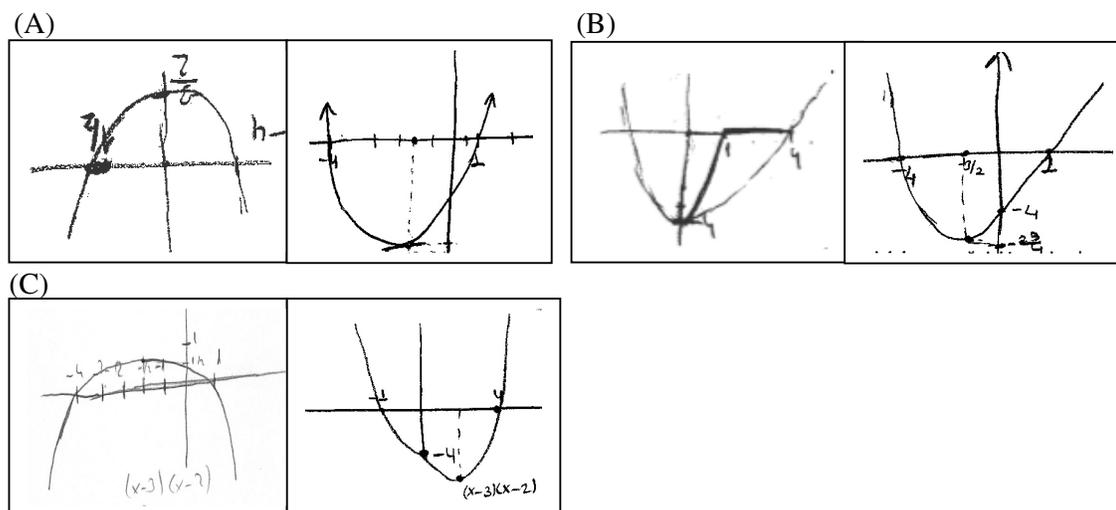


Figure 9. (A) The answer and correction of student number 1, (B) The answer and correction of student number 2, (C) The answer and correction of student number 4.

When the prospective teachers finished correcting the graphs, they proposed that the following changes would happen in the graphs related to changes in the equations:

- If we change the coefficient of x^2 into -3, the arms of the parabola will be downwards. However, the points where it crosses the axes do not change because other coefficient values stay the same. If we change the coefficient of x into -2, the points where the graph crosses the axes change. I believe it will also change when we change the constant because the equation changes.

- In my opinion, the points where the graph crosses the axes will change in all three cases. I saw that the arms of the parabola also changed in the drawings I did on GeoGebra. However, I cannot predict what kind of changes will happen here.

After some practice with GeoGebra, the prospective teachers made some corrections on the rational function graph in the last question and these corrections are presented in Figure 10. The first and the second versions of the graphs are given alongside to make the changes more noticeable.

As can be seen from the drawings, the prospective teachers could draw more accurate graphs after their practice with GeoGebra. However, the researchers agreed on a general conclusion that the concept of asymptote could not be totally comprehended. Thus, to identify how the prospective teachers visualized the concept of “asymptote” in their minds, the researchers asked the prospective teachers to answer the question: “What do you understand from the concept of asymptote?” The answers to this question were as follows:

- I know that the value which brings the denominator to the zero level is asymptote. However, I thought that I should take the expression as a whole, so I accepted the asymptote as $x-4$. Then during my practice with GeoGebra, I realized that this was a mistake.

- Because I used to know that graphs never crossed the asymptote, I did not continue the graph after the asymptote. But I saw that it can be divided into two or even 3 parts in some graphs. Because of this, I reviewed my drawing and saw that the graph continued after the asymptote.

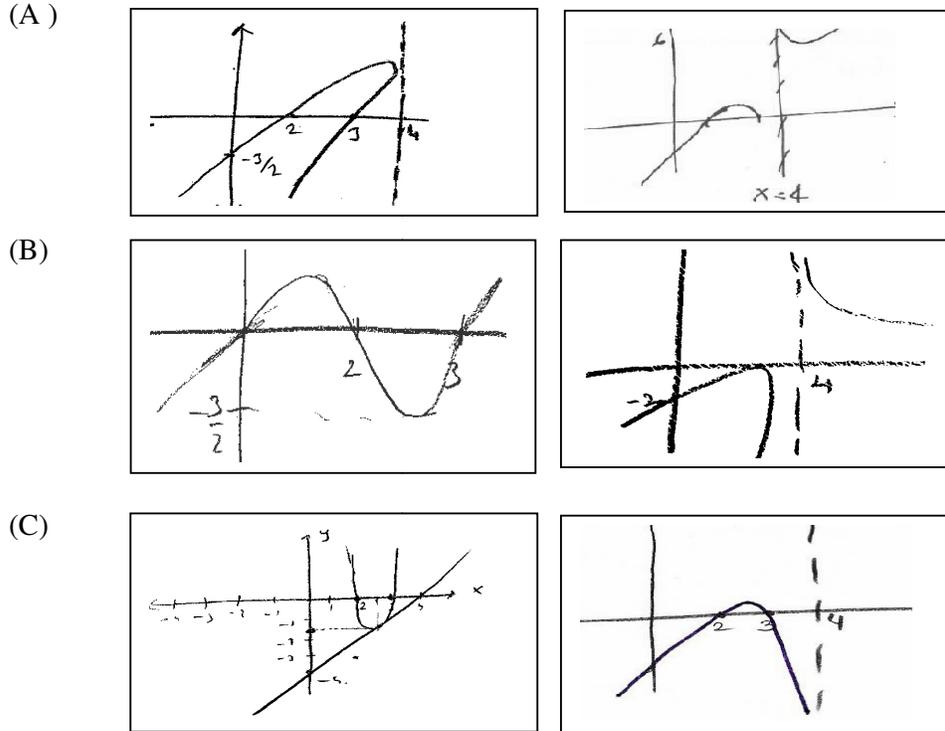


Figure 10: (A) The answer and correction of student number 2, (B) The answer and correction of student number 4, (C) The answer and correction of student number 5.

Findings Related to the GeoGebra Use Abilities of Prospective Teachers

The difficulties the prospective teachers faced while using this kind of dynamic geometry software were tried to be identified through various observation notes taken during their practice with GeoGebra. The first finding was that the prospective teachers had difficulty in adjusting to the environment of dynamic geometry software. Although the menu tools and their functions were circumstantially explained by the researchers before the application, the prospective teachers needed help at some different points during the practice.

The prospective teachers needed help when they were trying to write the equation in the equation bar. In this step, it was noticed that they had difficulty in writing exponential expressions (such as writing x^2 for x^2 symbol), defining the numbers in the multiplication form (writing $x*y$ for $x.y$), changing the features of the graph (such as expressing the units of the axes in natural numbers or in trigonometric expressions).

Findings Related to the Opinions of Prospective Teachers about Dynamic Geometry Software

The following are the questions asked by the researchers and the answers given by the prospective teachers during the focus group. (Researcher: R, Prospective Teachers: PT):

R- What do you think about GeoGebra software? What did you learn? Are you satisfied with this practice?

PT2- GeoGebra helps us to notice differences quicker while changing the given graph in the questions. Instead of being overwhelmed by calculations; instead of being busy with a lot of things...

PT5- At least, the difference between the change in the coefficient of x in $y = \sin x$ and the change in the coefficient of $\sin x$ is clear. In my opinion, this is a very useful program in this sense.

PT1- I think it is very good and useful. The menu and usage are very practical.

PT3- It was a good practice, but I could not see the graph clearly in some cases. It is good to see the graph and the changes that will happen in the graph roughly, but it does not give me much confidence for a detailed graph.

R- Well, what are your expectations from this kind of software? What kind of features do you want to see in addition to the ones here?

PT3- For example, the drawings can be made more precise.

R- What do you mean by "precise drawing"?

PT3- For example, it could calculate the breakpoints of different lines and show it to us on the graph.

PT2- I think every feature is efficient; there is no need to add anything. I haven't used a program like this before. As it is the first time, I think it is efficient enough. However, from now on I can try to do more things with this program.

PT4- I think if it was three-dimensional, the figures would be catchier. I think this feature is necessary. For instance, it would be more useful if I could rotate the figures and see them from different perspectives.

PT1- I think it is a very good program, and it can support our every need.

R- Which features did you like the most?

PT1- Well, when we draw the graph ourselves, we cannot be sure whether it is wrong or correct. In those cases, we can easily apply to this program.

R- Would you like to use this software in your lessons during your professional life in the future?

PT2- Yes, I can. These visual materials work well in lessons...for us to understand better.

PT3- Not only in mathematics, for example, I think it will help to understand the subjects better while doing experiments in physics, chemistry.

R- In which lessons do you think it would be more appropriate to use this program?

PT4- In my opinion, physics is really suitable for this because I think it will be appropriate to use it in graphs such as time-temperature graphs. Of course, it will be generally more useful in mathematics.

PT1- I agree, but as physics and mathematics work together, I think we can use this program easily.

Results and Conclusion

As a result of the analysis on prospective teachers' materials, it was noticed that they could easily draw the graphs of polynomial functions, but had difficulty in drawing the trigonometric and rational functions. In their studies, Kutluca and Baki (2009) emphasized that drawing the graphs of trigonometric functions was among the most difficult topics that high school students faced. In the light of these findings, it is possible to say that the weaknesses in graphing abilities may stem from previous learning experiences. Other results showed that the difficulties faced while drawing the graphs of rational functions arise from not being able to completely comprehend the concept of asymptote.

While reviewing the answers they gave to the questions after the practice, the prospective teachers were able to notice the mistakes and present faults with the help of GeoGebra. This conclusion was drawn because the answers given during the interviews included expressions like "I recognized that it would be in this way during my practice with GeoGebra". However, the aim is to correct the mistakes as well as recognize them in such applications. In this study, although this aim was partially achieved, it can be possible to bring the graphing abilities to higher levels with long-term applications. It is a well-known fact that prospective teachers have difficulty in interpreting the relationships between physical concepts and graphs (McDermott, Rosenquist & Van Zee, 1987). Thus, it is important to note that the graphing abilities of prospective physics teachers can be improved with the help of such software.

The results of the observations obtained during the application of GeoGebra on prospective teachers showed that they had difficulty in adapting to this kind of dynamic geometry software. During the observations, it was noticed that the prospective teachers had difficulty in finding the related menu tools when they were asked to make various changes in features after they had drawn the graphs. Therefore, it is important to emphasize that prospective teachers should have at least basic level computer abilities as well as know the software related to their departments to be able to use this software.

In the light of interviews conducted on the prospective teachers, it was observed that they were quite satisfied with the practice related to the teaching of the software. Being able to draw graphs, see the changes happening in the graphs in a short time without a need for any operation, and the fact that the program is easily accessible were noted among the reasons why the prospective teachers were pleased with this software. The prospective teachers emphasized especially the benefits of the visualization features of the software. They also proposed some suggestions about the addition of some features. These additional features suggested by the prospective teachers might be added with the use of more special and complex software (such as Maple, Mathematica, MatLab) in the following stages.

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