

## Discovery of Ambiguity in the Traditional Procedure of Handling Physical Quantities

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### Abstract

This paper is concerned with physics education at a basic level where one is to deal with mathematical symbols linked to scientific concepts. It aims at identifying the remarkable fact that the very manner in which the symbol of a physical quantity has been treated in the procedure of solution of formula-based numerical problems in physics at many places of the traditional literature (journals and textbooks) is ambiguous. Such a procedure does not take care of the simultaneous substitution of the unit of the symbol of a physical quantity along with its numerical value, thereby violating the fundamental concept of “Quantity calculus”, according to which a physical quantity (represented by a particular symbol) is the product of a numerical value and a unit. With a view to getting rid of the ambiguity as well as to bring precision and sophistication in the procedure of solution of formula-based numerical problems in physics, this paper therefore emphasizes the immediate need of considering simultaneous substitution of both the numerical value as well as the unit of each and every symbol of a physical quantity appearing in the relevant physical equation.

### Keywords

Physical quantity; Unit of physical quantity; Physical equation; Quantity equation

♦Received 28 December 2019 ♦Revised 21 May 2020 ♦Accepted 23 May 2020

### Introduction

“Teaching problem solving” is a topic falling within the purview of physics education. In this field, we are to make use of the appropriate law(s) of physics based on which the given problem could be solved. Solving numerical problems in physics based on a physical equation or the mathematical form of a law involving symbols of physical quantities forms an important part in the physics curriculum at all levels starting from the basic level where one is to deal with mathematical symbols linked to scientific concepts. That is why solution of such formula-based numerical problems is widespread in the scientific literature (journals and textbooks). This paper considers the traditional procedure of formula-based numerical problem solving in physics in particular and in different branches of science and engineering in general.

Hung and Wu (2018, p. 3) presented the procedure of solution of Question 3 in numerical problem for finding acceleration of Jill as follows.

$$\begin{aligned}F_{\text{Jack}} &= m_1 a_1 = 60 \times 3 = 180 \text{ (N)} \\F_{\text{Jill}} &= 180 = m_2 a_2 = 40 \times a' \\a' &= 4.5 \text{ (m/s}^2\text{)}\end{aligned}$$

Such a procedure of solution takes care of substituting only the numerical values of the physical quantities appearing in symbolic forms in the relevant physical equations violating the most fundamental fact that a physical quantity, in general, have got a numerical value along with its unit.

In addressing problem number 2, the author in (Martinez-Borrenquero, G., Pérez-Rodríguez, A. L., Suero-López, M. I., & Naranjo-Correa, F. L., 2018, p. e3401-3), made use of the formula  $u_{\text{rx}} = \frac{n_1}{n_2} u_{\text{ix}}$ , where  $u_{\text{ix}}$  and  $u_{\text{rx}}$  are the x-component of velocities of the incident light and refracted light respectively,  $n_1$  and  $n_2$  being the refractive indices of water and air respectively. Only the numerical values of the x-components of the velocities (i.e.  $u_{\text{ix}}$  and  $u_{\text{rx}}$ ) are substituted in the said formula without taking any care of the substitution of the unit of each of the two velocities  $u_{\text{ix}}$  and  $u_{\text{rx}}$  along with their numerical values in obtaining the final result (i.e. the minimum height  $h$ ).

The same type of procedure of substituting only the numerical values of concerned physical quantities in the calculation of “ $k$ ” has also been reflected from the following quoted lines of (Mitra, 2011, p. 120). “Numerical example 1: For a right circular cylindrical water tank let us take in the light of the above feature,  $b = 36$  cm,  $H = 100$  cm,  $r = 4$  cm,  $g = 980$  cm/s<sup>2</sup>, radius of the hole = 0.1 cm. Then  $k = \frac{a\sqrt{2g}}{\pi r^2} = \frac{\pi (1)^2}{\pi (4)^2} \sqrt{2 \times 980} = 28 \times 10^{-3}$  (approx)”. The author (Nair, 2017, p. 51) considered calculating the de Broglie wavelength for the earth as:

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{[(6 \times 10^{24}) \times (3 \times 10^4)]} = 3.6 \times 10^{-63} \text{ meter.}$$

Elsewhere, the equivalent resistance  $R$  of a 1 ohm and 200 ohm coil in parallel has been calculated as: “The equivalent resistance  $R$  of a 1 ohm and 200 ohm coil in parallel is given by

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{200} = 1.005, \text{ i.e. } R = 0.995 \text{ ohm} \text{ (Smith, 1947, p. 854).}$$

Dawes (1922, p. 23) considered calculation of electric power ( $P$ ) of the problem appearing at the said page as:

$$P = EI = 115 \times 30 = 3,450 \text{ watts.}$$

In another source (Yarwood, 1973, p. 57),

Bohr magneton ( $\mu_B = \frac{he}{4\pi m}$ ) has been calculated as:

$$\mu_B = \frac{6.625 \times 10^{-34} \times 1.76 \times 10^{11}}{4\pi} = 9.28 \times 10^{-24} \text{ Am}^2.$$

Furthermore, for dealing with the problem in an older source (Yorke, 1906, p. 109), work done has been calculated as: Work done by the current during the 600 *secs.* =  $E \times C \times T = 100 \times 0.46 \times 600 = 27,600 \text{ joules of work.}$

It would now be worth mentioning here that in each of the aforementioned procedures of calculation, only the numerical values of the concerned physical quantities appearing as symbols in the relevant formula have been substituted disregarding totally simultaneous substitution of the relevant unit of each and every physical quantity appearing in symbolic form along with its numerical value. The fact that the same type of approach is wide-spread in many scientific textbooks (Mathur, 1962; Starling, 1929; Hecht, 2017; Nelson & Parker, 1970; Partington, 1921; Shashol'skaya & El'tsin, 1963; Everest & Pohlmann, 2009; Saha & Srivastava, 1967; Nasar, 1998; Edminister, 1965; Robertson, 1955; Frye, 1947; Jamieson, 1910; Yorke, 1906; Slater, 1939; Waseda et al., 2011; Ayres, 1954; Timbie & Higbie, 1915; Edser, 1911; Bansal, 2009; Eldridge, 1940; Landau & Kitaigorodsky, 1980; Duncan & Starling, 1925; Benson, 1965; Savelyev, 1989; Jenkins & White, 1932; Sinclair & Dunton, 2007) has been detected by dint of an extensive search and that has also been shared with in this paper.

I noted the following assertions from above:

- Physical quantities, as far as their independent identities are concerned, have not been properly honored with in the procedure of solution of law or formula-based numerical problems in many traditional resources.
- Most of the traditional resources have given emphasis on substituting only the numerical value of the symbol of a physical quantity appearing in the relevant physical equation there by violating the fundamental fact that, as per "Quantity calculus" (Taylor, 2018), a physical quantity is to be expressed as the product of its numerical value and its unit. i.e. **physical quantity =  $n \times u$** , so that simultaneous substitution of the numerical value of a physical quantity in symbolic form as well as its unit is essential.

This issue considered in this paper is novel and that has never been tried earlier. The pertinent issue raised in this contribution is in respect of the misleading treatment of handling physical quantities while dealing with the procedure of solving formula-based numerical problems in physics. With a view to getting rid of the aforesaid ambiguous procedure of handling physical

quantities, this paper emphasizes the urgent need of incorporating both the numerical value as well as the unit of the relevant physical quantity at all places of mathematical manipulation involving such a physical quantity in symbolic form. The message conveyed vide this paper will bring precision, enhance and sophisticate the relevant field of study there by getting rid of the ambiguous concept prevailing so far in the traditional literature (journals and textbooks).

### Results of an extensive search of textbooks

This section reflects the result of an extensive search of the scientific literature with respect to the procedures adopted for solving law or formula-based problems with given numerical data.

#### Case 1:

No account has been taken care of in respect of substituting the units of the relevant physical quantities along with their numerical values in (Mathur, 1962, p. 271) while calculating the earth's surface potential ( $v$ ) using the formula  $v = \frac{GM}{x}$ , in the solved problem No. 8.

#### Case 2:

The following quotation prevailing in (Starling, 1929, p.164) clearly indicates that the units of relevant physical quantities have never been incorporated along with their numerical values in the formula used.

$$I_0 = \frac{E_0}{\sqrt{L^2 \omega^2 + R^2}} = \frac{100}{7.447} = 13.43 \text{ amperes}''$$

#### Case 3:

Attention may be given to the solution of Example 5.2 that appeared at (Hecht, 2017, p. 165) in which the units of  $s_0$  and  $R$  have never been considered while substituting their numerical values in the relation:  $\frac{n_1}{s_0} + \frac{n_2}{s_1} = \frac{n_2 - n_1}{R}$ , to obtain the unknown quantity  $s_1$  as:  $s_1 = 66.9 \text{ cm.}$

#### Case 4:

Consider the solved problem number 1 in (Nelson & Parker, 1970, p. 6) in which the relation  $v^2 = u^2 + 2as$  has been used to find  $s$  as:  $s = 43 \frac{3}{4} \text{ m}$  by substituting only the numerical values of  $u$ ,  $v$ , and  $a$  without incorporation of their units.

#### Case 5:

The solution procedure of Example 3 in (Partington, 1921, p. 68) makes use of the formula

$$v_2 = v_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1}$$

and  $v_2$  is obtained as:  $v_2 = 4.209 \text{ litres}$  by substituting only the numerical values of  $v_1$ ,  $P_1$ ,  $P_2$ ,  $T_1$ , and  $T_2$  without incorporation of the units of those physical quantities in the said formula.

**Case 6:**

Units of concerned physical quantities have never been substituted along with their numerical values in the equation considered in (Shashol'skaya & El'tsin, 1963, p. 86) for the solution of problem number 3.

**Case 7:**

The unit of frequency (viz. Hz) never appears in the relevant equations employed in the solution procedure of Example 4 appearing at (Everest & Pohlmann, 2009, p. 14) to arrive at the solutions for  $f_1$  and  $f_2$  as:  $f_1 = 1,767.8$  Hz, and  $f_2 = 3535.5$  Hz.

**Case 8:**

The numerical problem considered in (Saha & Srivastava, 1967, p. 15) makes use of the formulae (8) and (9) appearing at page number 14. But it can be readily verified that the units of relevant physical quantities have never been substituted along with their numerical values in those two formulae for the calculation of the result.

**Case 9:**

The numerical problem number 1.27 appearing at (Nasar, 1998, p. 3) has been solved as:  $U = Pt = 110 \times 0.9 \times 10^{-3} \times 12 \times 30 = 35.64$  kWh, based on which cost of operation has been found as \$ 2.50 without incorporating the units of  $P$  and  $t$  along with their numerical values in the formula  $U = Pt$ .

**Case 10:**

The solution of problem number 8.3 appearing at (Edminister, 1965, p. 88) makes use of the formula  $c = \frac{1}{L(2\pi f)^2}$  to obtain  $c$  by substituting only the numerical values of  $L$  and  $f$  without considering their relevant units along with the numerical values.

**Case 11:**

The formula  $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$  has been used in Example 2 appearing at (Robertson, 1955, p. 52) to obtain  $n$  as:  $n = 1.5$ . But in the procedure of solution of this problem, only the numerical values of relevant physical quantities without their units have been substituted in the said formula to obtain the above result.

**Case 12:**

The second illustrative example of Art. 3-8 appearing at (Frye, 1947, p. 18) makes use of the formula for kinetic energy of the ball as  $\frac{wv^2}{2g}$  and by taking  $w = 1.00$  pound,  $v = 100$  feet/second and  $g = 32.3$  feet/second<sup>2</sup>, the kinetic energy has been calculated by substituting only the numerical values of the concerned physical quantities without any incorporation of their respective units as: kinetic energy =  $(100)(100)^2/(2)(32.2)$  or 155.3 foot-pounds.

**Case 13:**

The problem considered in EXAMPLE III appearing at (Jamieson, 1910, p. 8) has been solved by using the relation:

Work done = whole weight of the beam  $\times$  height through which its c. g. is raised

and substituting only the numerical value of the concerned physical quantities without incorporation of their relevant units.

**Case 14:**

The following quoted lines from (Slater, 1939, p. 61) are now being considered.

“ $k = \frac{R}{N_0}$  or  $R = N_0 k$ , so that  $k = \frac{8.314 \times 10^7}{6.03 \times 10^{23}} = 1.379 \times 10^{-16}$  erg per degree”

From this calculation, it follows that no attention has been given in respect of the simultaneous incorporation of the unit of the physical quantity  $R$  along with its numerical value while obtaining the physical quantity  $k$ .

**Case 15:**

Question 1.1 appearing at (Waseda et al., 2011, p. 6) is as follows.

Calculate the energy released per carbon atom when 1 g of carbon is totally converted to energy.

The solution (Answer 1.1) of the said problem appearing at the same page makes use of the formula  $E = mc^2$ , where  $m$  is mass and  $c$  is the speed of light to obtain  $E$  as:

$$E = 1 \times 10^{-3} \times (2.998 \times 10^{10})^2 = 8.99 \times 10^{13} \text{ J}$$

It can therefore be readily seen from above that the units of  $m$  and  $c$  have never been substituted along with their numerical values in the aforesaid formula to find  $E$ .

**Case 16:**

The following quoted lines from (Ayres, 1954, p. 22) are in connection with the solution of problem number 13.

“ $\cot A = \frac{AC}{CB}$  and  $AC = CB \cot A = 120 \cot 15^\circ = 120 (3.7) = 444 \text{ ft}$ ”

It is to be noted here that the incorporation of the unit of  $CB$  (which is ft in this problem as per question) along with its numerical value 120 has not been made in the above calculation.

**Case 17:**

Example 6 prevailing in (Timbie & Higbie, 1915, p. 67) is concerned with the following problem.

“What impedance has a circuit through which 110 volts alternating e.m.f. is able to force 5 amperes?”

This problem has been solved in the said page of (Timbie & Higbie, 1915) as follows:

$$Z = \frac{E}{I} = \frac{110}{5} = 22 \text{ ohms}$$

It can be readily seen that only the numerical values of  $E$  and  $I$  have been substituted in the above formula without incorporation of their respective units to obtain the impedance as 22 ohms.

**Case 18:**

Force ( $f$ ) in the second problem appearing at (Edser, 1911, p. 19) has been calculated by using the formula  $f = \frac{vm}{t}$  and the relevant calculation has been shown in the said page of (Edser, 1911) as follows:

$$f = \frac{39.5 \times 535}{2} = 10,600 \text{ dynes (nearly)}$$

It can be readily seen that the units of the relevant physical quantities have never been considered along with their numerical values in the aforesaid substitution for obtaining  $f$ .

**Case 19:**

Attention is now being drawn to the following quoted lines from (Bansal, 2009, p. 10) in connection with the solution of problem 1.2

$$\text{“Now Stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$95 = \frac{4000}{\frac{\pi}{4} D^2} = \frac{4000 \times 4}{\pi D^2}$$

$$\text{or } D^2 = \frac{4000 \times 4}{\pi \times 95} = 53.61$$

$$\text{or } D = 7.32 \text{ mm.}”$$

It can be readily seen from the above calculation that the units of the concerned physical quantities have not been considered while substituting the numerical values of those quantities in the aforesaid formula for obtaining  $D$ .

**Case 20:**

Example 2 found in (Eldridge, 1940, p. 232) is:

Find the change in pressure when a liter of gas, originally at a pressure of 70 cm of mercury, is heated from 27°C to 227°C and volume is halved.

The procedure of solution of this problem as found in the aforesaid page of (Eldridge, 1940) is now being reproduced below.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{70 \times 1}{300} = \frac{P_2 \times \frac{1}{2}}{500},$$

$$P_2 = 233 \text{ cm of mercury}$$

It is easy to find from above that only the numerical values of the relevant physical quantities have been substituted without any incorporation of their units in the above formula to find  $P_2$  as  $P_2 = 233$  cm of mercury in the above solution process.

**Case 21:**

Quoted lines from (Landau & Kitaigorodsky, 1980, p. 70) are:

“Consequently,  $g = \frac{v^2}{R}$ , from which we find the speed of satellite’s orbital motion:

$$v = \sqrt{gR} = \sqrt{8.9 \times 6.6 \times 10^6} = 7700 \text{ m/s} = 7.7 \text{ km/s}”$$

Here also it can be seen that the units of the concerned physical quantities are not considered along with their numerical values while substituting in the aforesaid formula for finding  $v$ .

**Case 22:**

The following quoted lines appearing at (Duncan & Starling, 1925, p. 692) are in connection with the calculation of velocity of sound in air at 0°C using the formula  $v = \sqrt{\frac{p}{d}}$ .

“For air at 0°C,  $p = 13.6 \times 76 \times 981$  dynes per sq. cm,  $d = 0.00129$  grams per c.c.;

$$\therefore \text{Velocity of sound} = \sqrt{\frac{13.6 \times 76 \times 981}{0.00129}} = 28100 \text{ cm. per sec.}”$$

In this calculation, no care has been taken about the substitution of the units of concerned physical quantities along with their numerical values.

**Case 23:**

A part of the solution procedure of problem number 148 prevailing in (Benson, 1965, p. 201) is quoted below.

$$\text{“Without feedback gain } A = \frac{\mu R_L}{r_a + R_L} = \frac{1000 \times 200 \times 10^3}{(200 + 200) \times 10^3} = 500\text{”}$$

It follows from the aforesaid calculation that only the numerical values of the concerned physical quantities have been given top priority in the substitution process discarding totally simultaneous substitution of their respective units along with.

**Case 24:**

Quoted lines prevailing in (Savelyev, 1989, p. 300) are:

“Introducing these values into Eq. (14,70), we get

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.40 \times 8.31 \times 290}{29 \times 10^{-3}}} = 340 \text{ m/s}”$$

Only the numerical values of physical quantities have been considered disregarding their respective units in the aforesaid calculation.

**Case 25:**

The example problem appearing at (Jenkins & White, 1932, p. 68) has been solved by making use of equation 4d (which is:  $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ ). A part of the solution procedure found at the said page of (Jenkins & White, 1932) is quoted below.

“Substitution of the known quantities in equation 4d gives

$$\frac{1}{25} = (1.520 - 1)\left(\frac{1}{\infty} - \frac{1}{r_2}\right)$$

$$\text{Transposing and solving for } r_2, \frac{1}{25} = 0.520 \left(0 - \frac{1}{r_2}\right) = -\frac{0.520}{r_2}$$

$$r_2 = -(25 \times 0.520) = -13.0 \text{ cm}”$$

One can readily see from above that the units of concerned physical quantities have not been considered along with their numerical values while substituting in the formula to obtain  $r_2$ .

**Case 26:**

Again quoted lines found in (Sinclair & Dunton, 2007, p. 4) are:

“For example, if a given length of a sample wire has a resistance of 12 ohms and its diameter is 0.3 mm, the same length of wire made from the same material but with a diameter of 0.4 mm will have resistance  $R$  given by:

$$R \times 0.4^2 = 12 \times 0.3^2$$

$$\text{So, } R = \frac{12 \times 0.3^2}{0.4^2} = \frac{12 \times 0.09}{0.16} = 6.75 \text{ ohms}”$$

In the aforesaid solution process only the numerical values of the concerned physical quantities have been considered during substitution and the incorporation of their units along with the numerical values have been totally ignored.

**Conclusion**

The target audiences for this paper are the novice students as well as their teachers at a basic level where one is to deal with mathematical symbols linked to scientific concepts. It considers the procedure of solution of formula-based numerical problems in physics. After going through an extensive search of textbooks (Mathur, 1962; Starling, 1929; Hecht, 2017; Nelson & Parker, 1970; Partington, 1921; Shashol'skaya & El'tsin, 1963; Everest & Pohlmann, 2009; Saha & Srivastava, 1967; Nasar, 1998; Edminister, 1965; Robertson, 1955; Frye, 1947; Jamieson, 1910; Yorke, 1906; Slater, 1939; Waseda et al., 2011; Ayres, 1954; Timbie & Higbie, 1915; Edser, 1911; Bansal, 2009; Eldridge, 1940; Landau & Kitaigorodsky, 1980; Duncan & Starling, 1925; Benson, 1965; Savelyev, 1989; Jenkins & White, 1932; Sinclair & Dunton, 2007), it has been detected that the very manner in which the symbol of a physical quantity involved in a physical equation has been treated in such a procedure of solution of formula-based numerical problems in physics is ambiguous. Such a procedure considers only the substitution of the numerical value of the symbol of a physical quantity present in the relevant physical equation without considering simultaneous substitution of the unit of the physical quantity along with its numerical value, thereby violating the fundamental concept of “Quantity calculus” (Taylor, 2018), according to which a physical quantity could be written as:

$$\text{Physical quantity} = \text{Numerical value} \times \text{Unit}$$

In order to get rid of such an ambiguity as well as to bring precision and sophistication in the procedure of solution of formula-based numerical problems in physics, this paper emphasizes immediate need of getting rid of this type of ambiguous approach for mathematical handling of physical quantities in the procedure of solution of formula-based numerical problems in physics by considering simultaneous substitution of both the numerical value as well as the unit of each and every symbol of a physical quantity appearing in the relevant physical equation as has been adopted in (Gartenhaus, 1977; Halliday, 1955; Halliday et al., 2011), although none of them takes care of the incorporation of any discussion regarding “Quantity calculus” (Taylor, 2018),

which is essential for building up a strong foundation about physical quantities and their mathematical handling.

In view of above, the present work is of paramount importance to the novice students and their teachers in particular, and to the physics education community in general. The merits of the present scheme must be judged in the following.

- (i) The pertinent issue raised in this contribution is entirely novel and has never been tried earlier.
- (ii) Implementation of the proposal offered will remove the ambiguity present in the procedure of solution of formula-based numerical problems in physics prevailing at many long-used physics literature (journals and textbooks).
- (iii) The proposed scheme will make the procedure of solution of law-based numerical problems in physics totally unambiguous and it will bring precision and sophistication in the relevant field of study there by enhancing the same as well.

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